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Location optimization of multiple distribution centers under fuzzy environment*

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Abstract: Locating distribution centers optimally is a crucial and systematic task for decision-makers. Optimally located distribution centers can significantly improve the logistics system's efficiency and reduce its operational costs. However, it is not an easy task to optimize distribution center locations and previous studies focused primarily on location optimization of a single distribution center. With growing logistics demands, multiple distribution centers become necessary to meet customers' requirements, but few studies have tackled the multiple distribution center locations (MDCLs) problem. This paper presents a comprehensive algorithm to address the MDCLs problem. Fuzzy integration and clustering approach using the improved axiomatic fuzzy set (AFS) theory is developed for location clustering based on multiple hierarchical evaluation criteria. Then, technique for order preference by similarity to ideal solution (TOPSIS) is applied for evaluating and selecting the best candidate for each cluster. Sensitivity analysis is also conducted to assess the influence of each criterion in the location planning decision procedure. Results from a case study in Guiyang, China, reveals that the proposed approach developed in this study outperforms other similar algorithms for MDCLs selection. This new method may easily be extended to address location planning of other types of facilities, including hospitals, fire stations and schools.

Key words: Multiple distribution centers, Location selection, Clustering algorithm, Axiomatic fuzzy set (AFS), Technique for order preference by similarity to ideal solution (TOPSIS)
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1 Introduction

Distribution centers play a vital role in transportation and logistics systems. With growing urban freight movements, a single distribution center in a large region cannot efficiently accommodate the growing demands from customers due to lengthened transit time and high warehouse handling cost, especially for just-in-time and cold chain deliveries. Therefore, systems with multiple distribution centers become highly desirable in practice and locating these centers optimally is a critical issue faced by logistics system operators and transportation planners (Yang *et al*., 2007). A method for optimal location selection of multiple distribution centers helps balance production and consumption, which further improves the efficiency in logistics systems, optimizes distribution system networks, and alleviates urban traffic congestion. However, evaluating and selecting appropriate distribution center locations is not a straightforward task because many factors and criteria need to be taken into account in the decision-making procedure (Syam, 2002; Sun *et al*., 2008). For example, distance to the shared production facility, local traffic pattern, and economic condition for each potential distribution center, all need to be considered. These factors

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require certain domain knowledge to justify and should be adequately quantified and incorporated into the multiple locations selection problem. It is worth mentioning that a general solution to the problem of multiple distribution center locations (MDCLs) selection problem also benefits the planning of other types of multiple facility locations such as hospitals, fire stations, and schools.

Most previous studies focused on selecting a single distribution center location rather than simultaneously selecting MDCLs. The two most classical quantitative frameworks for single facility location selection are the cost minimization approach (Hansen *et al*., 1992; Lee, 1993; Tyagi and Das, 1995) and the profit maximization approach (Hakimi and Kuo, 1991). Yang *et al*. (2007) constructed a mathematical model to minimize the total relevant cost, including setup cost, turnover cost and transportation cost with each customer's demand. Sun *et al*. (2008) presented a bi-level mixed-integer programming model to seek the optimal distribution center location, where the upper-level model minimized the planners' cost, and the lower-level model minimized the customers' cost. Küçükaydin *et al*. (2011) also formulated a bi-level mixed-integer nonlinear programming model to deal with a competitive facility location problem based on the game theory, which maximized the profit and the attractiveness of each facility simultaneously. Cost minimization and profit maximization approaches mainly focus on increasing the revenue for logistics operators. However, there are other critical factors that should be taken into account in the MDCL problem, such as natural and business environments, supply condition, etc. Both cost minimization and profit maximization approaches fail to capture those factors.

These aforementioned influential factors are traditionally measured by human perception, and are considered fairly difficult to process by computers in an automatic fashion. To tackle the vagueness of selection criteria, the fuzzy set theory has been widely used in decision-making under ambiguous conditions (Zadeh, 1965; Chen, 2001; Chou *et al*., 2008; Awasthi *et al*., 2011). One of the main advantages with fuzzy theory is that it is capable of not only considering uncertainty in human's cognitive process, but also evaluating different subjective attributes by linguistic terms (Liang and Wang, 1991; Chu, 2002). The fuzzy set theory has been applied in the facility location selection process. Since evaluation criteria for each alternative location are often defined in linguistic terms, e.g., "Low", "Medium", "High", "Poor", "Fair", "Good", etc., the fuzzy set theory is well suited to transform these linguistic attributes into fuzzy relations, fuzzy numbers, and fuzzy inference systems (Chen, 2001; Lee and Donnell, 2007; Lin and Ke, 2009). Thus, there are many facility location selection studies that employ the fuzzy set theory to interpret evaluation criteria such as labor costs, resource availability, freight regulation, etc. (Kahraman *et al*., 2003).

The fuzzy set theory has also been applied to the location ranking process. Chou (2009) proposed an integrated short- and long-term multiple-criteria decision-making approach to solve location selection problems. Wey and Chang (2009) presented a hybrid analytical hierarchy process (AHP) and a data envelopment analysis (DEA) location ranking method. Yu *et al.* (2011) ranked and evaluated the potential location plans for multiple urban transit hubs by using an AHP-based framework and fuzzy logic. Among all location ranking algorithms, the technique for order preference by similarity to ideal solution (TOPSIS) algorithm proposed by Negi (1984) is most widely used and has been improved by many researchers. Chu (2002) presented a fuzzy TOPSIS model under a group of decisions to solve the facility location selection problem. Awasthi *et al*. (2011) also used the fuzzy TOPSIS approach to evaluate and select the best urban distribution center. However, traditional fuzzy evaluation approaches are primarily used to select a single facility location, and most of them are hard to expand to tackle the MDCL problem, since the traditional fuzzy evaluation approaches are not able to simultaneously consider multiple heterogeneities among different distribution center locations, such as local economic condition, population density, and geographic position, etc. The selected distribution center locations may be significantly biased. For instance, distribution centers may not be selected in those densely populated but poverty-stricken areas.

To fill this research gap and provide a potential solution for the MDCL problem, a novel approach for MDCLs selection is proposed. The remaining parts of this paper are organized as follows: comprehensive evaluation index system is first established by using fuzzy integration and clustering algorithms, which groups several potential locations into different clusters. Then the fuzzy TOPSIS method is applied for selecting the alternative distribution center locations within each cluster. Next, a case study of MDCL in Guiyang, China is presented to demonstrate the effectiveness of our method, followed by a comparison between different relevant algorithms. Last and but not least, a sensitivity analysis is conducted to examine impacts incurred by changing selection criteria weights. Finally, conclusions are given at the end of this paper.

2 Methodology

In this paper, the study methodology contains three parts: (1) major criteria and sub-criteria are established to form a hierarchical analysis structure; (2) the linguistics variables used for location evaluation are then defined and transformed into fuzzy numbers for further assessments; and (3) a location planning framework for urban multiple distribution centers is finally developed based on the fuzzified evaluation criteria.

2.1 Hierarchical analysis structure for location planning

The location selection criteria are used for evaluating potential urban distribution center locations. As shown in Fig. 1, six major criteria (C_i) and 16 sub-criteria (C_{ij}) are selected to determine the locations to implement urban distribution centers. These criteria are obtained from previous studies (Chen, 2001; Kahraman *et al.*, 2003), and are considered important after discussions with several decision-makers and members in the local transportation agencies.

These experts gave their linguistic evaluations on each sub-criterion. Based on these evaluations, each potential location can be evaluated and graded using linguistic terms by the experts. The score of an upper level criterion can then be calculated from these sub-criteria evaluations.

The above sub-criteria structure can be divided into two categories: cost criteria and benefit criteria. Sub-criteria C_{43}, C_{61} and C_{62} fall into the cost criteria, where an alternative location with higher cost is less likely to be selected as the final location. The remaining sub-criteria are grouped as benefit criteria, where an alternative location with higher benefit criteria is more likely to become the final location.

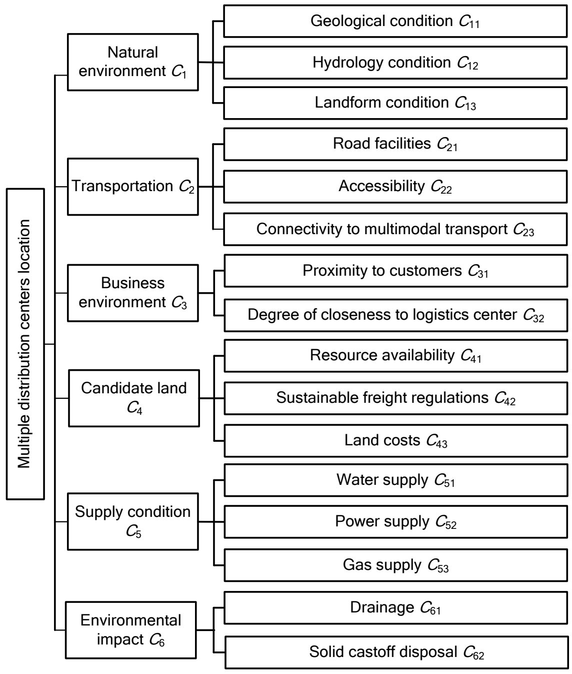

Fig. 1 Hierarchical analysis structure for location selection of multiple distribution centers

2.2 Linguistic variables fuzzification and related definitions

2.2.1 Linguistic variables fuzzification

As the first step, natural language is converted into numerical inputs using the fuzzy set theory. Adopted in this study is the triangle fuzzy transformation, in which a triangular fuzzy number is represented as a triplet $\tilde{n}=(n_1,n_2,n_3)$ (Buckley, 1985; Awasthi *et al.*, 2011; José *et al.*, 2012), and the membership function $\mu_{\tilde{n}}(x)$ can be calculated based on the triangular fuzzy number $\tilde{n}$ as follows:

$$\mu_{\tilde{n}}(x)=\begin{cases}0, & x\le n_1,\\ \dfrac{x-n_1}{n_2-n_1}, & n_1\le x\le n_2,\\ \dfrac{n_3-x}{n_3-n_2}, & n_2\le x\le n_3,\\ 0, & x\ge n_3,\end{cases} \tag{1}$$

where n_1, n_2, n_3 are real numbers, x at n_2 gives the maximal grade of $\mu_{\tilde{n}}(x)$, and x at n_1 and n_3 gives the minimal grade of $\mu_{\tilde{n}}(x)$.

In the fuzzy set theory, each linguistic term is transformed into a fuzzy number. Due to the uncertain nature of the MDCL problem, criteria ratings and alternative ratings are used as the linguistic variables (Wang and Qian, 2007). In the current study, as shown in Table 1, we apply a scale of 1–9 (VL–VH) to rate the criteria and (VP–VG) to rate the alternative locations. Table 1 presents the linguistic variables and fuzzy numbers for each criterion and alternative.

2.2.2 Related definitions

To address the MDCL selection problem, several related definitions are needed and presented as follows.

Definition 1 Notations for the parameters are defined as follows:

$D\{D_i\,|i=1,2,\cdots,m\}$ denotes the decision-makers who evaluate the alternative distribution center locations, and m is the total number of decision-makers;

$C^1\{C_t^1\,|t=1,2,\cdots,r\}$ denotes the major criteria for location selection of multiple distribution centers, and r is the total number of major criteria;

$C^2\{C_t^2\,|t=1,2,\cdots,k\}$ denotes the sub-criteria for location selection of multiple distribution centers, and k is the total number of sub-criteria;

$A\{A_i\,|i=1,2,\cdots,n\}$ is the potential locations of urban distribution centers, and n is the total number of potential locations;

$A'_h\{A'_{hi}\,|i=1,2,\cdots,n';h=1,2,\cdots,c\}$ is the number of potential locations within cluster h, and c is the total number of final clusters.

$W_{ut_l}^2\,(u=1,2,\cdots,m;t=1,2,\cdots,r)$ denotes the fuzzy numbers for sub-criterion l of major criterion t by decision-maker u;

$W_{ut'}^2\,(u=1,2,\cdots,m;t'=1,2,\cdots,k)$ denotes the fuzzy numbers for sub-criterion t' by decision-maker u;

$X_{uhit'}^2\,(u=1,2,\cdots,m;\ h=1,2,\cdots,c;\ i=1,2,\cdots,n';\ t'=1,2,\cdots,k)$ denotes the fuzzy numbers for alternative location i under sub-criterion t' by decision-maker u within cluster h;

$x_{ujt_l}^2\,(u=1,2,\cdots,m;\ j=1,2,\cdots,n;\ t=1,2,\cdots,r)$ denotes the fuzzy numbers for alternative location j under sub-criterion l of major criterion t by decision-maker u;

$p'\{p'_h\,|h=1,2,\cdots,c'\}$ represents each initial cluster, and c' is the total number of initial clusters.

$p\{P_h\,|h=1,2,\cdots,c\}$ represents each final cluster.

Definition 2 $X=\{x_1,x_2,\cdots,x_n\}\subseteq R^n$ denotes the sample set of potential urban distribution center locations; $\mu=\{\mu_1,\mu_2,\cdots,\mu_r\}$ is the attribute set of X; $x_{tj}=\mu_t(x_j)$, $t=1,2,\cdots,r$, $j=1,2,\cdots,n$, where x_{tj} denotes the membership function value of the sample x_j with attribute μ_t. The attribute μ_t can be further separated as $m_{t,1},m_{t,2},\cdots,m_{t,s_t}$, thereby, the attribute set μ can be expressed as $\mu=M=\{m_{1,1},m_{1,2},\cdots,$

Table 1 Linguistic terms for criteria and alternative ratings

Linguistic term	Abbreviation	Fuzzy number
Very low (very poor)	VL (VP)	(0.56, 0.62, 0.65)
Between very low and low (between very poor and poor)	B.VL&L (B.VP&P)	(0.62, 0.65, 0.68)
Low (poor)	L (P)	(0.65, 0.68, 0.73)
Between low and medium (between poor and fair)	B.L&M (B.P&F)	(0.72, 0.75, 0.78)
Medium (fair)	M (F)	(0.75, 0.78, 0.83)
Between medium and high (between fair and good)	B.M&H (B.F&G)	(0.80, 0.83, 0.90)
High (good)	H (G)	(0.85, 0.88, 0.93)
Between high and very high (between good and very good)	B.H&VH (B.G&VG)	(0.90, 0.93, 0.96)
Very high (very good)	VH (VG)	(0.95, 0.98, 1.00)

$m_{1,s_1}, m_{2,1}, m_{2,2}, \cdots, m_{2,s_2}, \cdots, m_{r,1}, m_{r,2}, \cdots, m_{r,s_r}\}$, which is defined as the fuzzy attribute (concept) set.

Definition 3 Let X, M be two sets, and 2^M be the power set of $\tau: X \times X \to 2^M$, if τ satisfies the following conditions (Liu, 1998a),

$$AX1: \forall (x_1, x_2) \in X \times X, \tau(x_1, x_2) \subseteq \tau(x_1, x_1),$$
$$AX2: \forall (x_1, x_2), (x_2, x_3) \in X \times X,$$
$$\tau(x_1, x_2) \cap \tau(x_2, x_3) \subseteq \tau(x_1, x_3).$$

(M, τ, x) is called an axiomatic fuzzy set (AFS) structure, X is called a sample set, M is called an attribute (concept) set, and τ is called structure.

Definition 4 Let R be the binary relation of the sample set X (Zhang *et al.*, 2004a), when $x, y \in X, x \neq y$, R satisfies the following conditions:

(1) If $(x, y) \in R$, then $(x, x) \in R$;

(2) If $(x, x) \notin R$ and $(y, y) \in R$, then $(y, x) \in R$;

(3) If $(x, y) \in R$ and $(y, z) \in R$, then $(x, z) \in R$;

(4) If $(x, x) \in R$ and $(y, y) \in R$, then $(x, y) \in R$ or $(y, x) \in R$.

R is called the sub-preference relation, the corresponding concept of sub-preference relation is called simple concept, otherwise R is called complex concept.

Definition 5 Let m be the simple fuzzy concept, and $m \in \tau(x, y)$, $\rho_m: X \to R^+ = [0, \infty)$ (Zhang *et al.*, 2004a),

(1) $\rho_m(x) = 0 \Leftrightarrow (x, x) \neq R_m, x \in X$;

(2) $(x, y) \in R_m \Rightarrow \rho_m(x) \geq \rho_m(y), x, y \in X$; ρ_m is called the membership function of the simple fuzzy concept m.

Definition 6 Let $\eta(x_j)$ be the simple fuzzy concept set, that is, $\eta(x_j)=\{m_{t,s'} | t = 1, 2, \cdots, r;\ s' = 1, 2, \cdots, s_t;\ j = 1, 2, \cdots, n\}$, and let ζ_{A_j} be the fuzzy description of each sample A_j then, ζ_{A_j}=argmin$\{\prod m_{t,s'} \mid m_{t,s'} \in \eta(x_j),\ t = 1, 2, \cdots, r;\ s' = 1, 2, \cdots, s_t;\ j = 1, 2, \cdots, n\}$.

Definition 7 Let X and M be two sets, and let (M, τ, x) be an AFS structure (Liu, 1998a), i.e., $A \subseteq X$, $B \subseteq M$, then $\overline{B}(A) = \{y \mid y \in \rho_m(x), m \subseteq B, \forall x \in A\}$, where $\overline{B}(x)$ denotes the samples belonging to the degree of B is less than or equal to the sample x belonging to the degree of B.

Definition 8 Let X be a sample set, and $X \subseteq R^n$, $\overline{m} \subseteq \overline{B}$, then $L_m(x)$ is a measure of sample x belonging to the simple fuzzy concept m, and $L_m(x)$ is defined as

$$L_m(x) = \frac{\sum_{x \in \overline{m}(x)} \rho_m(x)}{\sum_{x \in X} \rho_m(x)}. \tag{2}$$

Definition 9 Let the fuzzy concept $B \subseteq M$, the membership function $\mu_B(x)$ of fuzzy concept B is defined as

$$\mu_B(x) = \inf_{m \in B}(L_m(x)) \in [0,1], \quad x \in X. \tag{3}$$

Definition 10 Let X be the sample set of potential urban distribution center locations, M be the simple fuzzy concept set of X, $B \subseteq M$, and n be the sample size, the membership information entropy (Shannon, 2001) function $E(B)$ and the membership distribution coefficient function $D(B)$ can be defined as

$$E(B) = -\sum_{x \in X} [\mu_B(x) \lg(\mu_B(x))], \tag{4}$$

$$D(B) = -\left[\left(\sum_{x \in X} \mu_B(x)/n\right) \lg\left(\sum_{x \in X} \mu_B(x)/n\right)\right]. \tag{5}$$

When $E(B)$ becomes smaller, the membership degree of the sample x within concept B is more approaching the two ends of the interval [0, 1], and the boundaries are clearer. When $D(B)$ is smaller, the membership degree of the sample x within concept B is more approaching one end of the interval [0, 1], instead of both left and right ends. Thereby, in order to comprehensively evaluate the membership information entropy and distribution coefficient, the evaluation index V can be defined as $V = E(B)/D(B)$, where when V is smaller, it is more reasonable that the concept B describes the sample X.

2.3 Proposed framework for location planning

There are three major steps included in this framework. Each evaluation sub-criterion should be properly mapped into the higher hierarchical criterion based on the fuzzy integration method in the first step, and then clustering algorithm is undertaken to group the possible alternative locations into different clusters. The last step is to finalize the final location within each cluster by using the fuzzy TOPSIS approach. These steps are presented in detail as follows.

2.3.1 Fuzzy integration method based on sub-criteria

$x^2_{ujt_l}$ and $W^2_{ut_l}$ are expressed as triangular fuzzy numbers: $x^2_{ujt_l}=(a^2_{ujt_l},b^2_{ujt_l},c^2_{ujt_l})$, and $W^2_{ut_l}=(h^2_l,g^2_l,k^2_l)$, respectively. The comprehensive evaluation index from all decision-makers for location j under major criterion t can be expressed as

$$Z^1_{tj}=\frac{1}{m\times t_s}\otimes\sum_{u=1}^{m}[(x^2_{ujt_1}\otimes W^2_{ut_1})\oplus\cdots\oplus(x^2_{ujt_s}\otimes W^2_{ut_s})], \quad (6)$$

where ⊗ denotes vector multiplication, and ⊕ denotes vector addition. Z^1_{tj} denotes the comprehensive triangular fuzzy number to evaluate the location j under major criterion t, m denotes the number of decision-makers, t_s denotes the number of sub-criteria s under major criterion t.

Let $Y=(a,b,c)$ be a triangular fuzzy number. The representation of triangular fuzzy number Y is $P(Y)=\frac{1}{4}(a+2b+c)$ (Heilpern, 1997; Li, 1999; Chen, 2001; Kahraman *et al.*, 2003; Chou *et al.*, 2008; Chou, 2009). Thereby, if we suppose that $Z^1_{tj}=(T^1_{tj},Q^1_{tj},H^1_{tj})$ denotes the triangular fuzzy number, then the integrated membership function value μ^1_{tj} of the location j under major criterion t will be described as

$$\mu^1_{tj}=\frac{1}{4}(T^1_{tj}+2Q^1_{tj}+H^1_{tj}). \quad (7)$$

2.3.2 Clustering algorithm procedure

The integrated membership function value μ^1_{tj} calculated from the fuzzy integration method is used as the input. Our next step is to conduct clustering analysis to divide the potential distribution centers into different clusters. The AFS theory logic (Liu, 1998b; Zhang *et al.*, 2004b) has been proven as an effective approach to tackle human perception related clustering problems. Since the traditional AFS method is not appropriate to incorporate a large number of criteria (Liu *et al.*, 2005), an improved AFS theory logic algorithm is detailed as follows.

Step 1: According to Definition 2, the integrated membership function value μ^1_{tj} can be transformed into three values. They are expressed as $\rho_{m_{t,1}}$, $\rho_{m_{t,2}}$, and $\rho_{m_{t,3}}$, respectively, and if we define $\mu_t(x_j)=\mu^1_{tj}$, then the membership function corresponding to each membership function value μ^1_{tj} can be written as

$$\begin{aligned}
&\rho_{m_{t,1}}(x_j)=\mu_t(x_j),\\
&\rho_{m_{t,2}}(x_j)=h_{t,1}-\mu_t(x_j),\\
&\rho_{m_{t,3}}(x_j)=h_{t,3}-\left|\mu_t(x_j)-h_{t,2}\right|,\\
&h_{t,1}=\max\left\{\mu_t(x_1),\mu_t(x_2),\cdots,\mu_t(x_n)\right\},\\
&h_{t,2}=\frac{\mu_t(x_1)+\mu_t(x_2)+\cdots+\mu_t(x_n)}{n},\\
&h_{t,3}=\max_{1\le i\le n}\left\{\left|\mu_t(x_j)-h_{t,2}\right|+h_{t,2}\right\},\\
&t=1,2,\cdots,r,\ n \text{ is the number of potential locations.}
\end{aligned} \quad (8)$$

Step 2: Calculate the fuzzy attributes of each sample.

Step 2.1: Calculate $L_{m_{t,1}}(x_j)$, $L_{m_{t,2}}(x_j)$, and $L_{m_{t,3}}(x_j)$ through Eq. (2) and Step 1.

Step 2.2: Define $\mu_{m_{t,1}}(x_j)=L_{m_{t,1}}(x_j)$, $\mu_{m_{t,2}}(x_j)=L_{m_{t,2}}(x_j)$, and $\mu_{m_{t,3}}(x_j)=L_{m_{t,3}}(x_j)$, and the maximum of membership fuzzy values under simple fuzzy concept μ_t can be expressed as

$$\eta_{t,s'}(x_j)=\left\{m_{t,s'}\left|\mu_{m_{t,s'}}(x_j)=\rho_{m_{t,s'}}(x_j)=\max\left\{\mu_{m_{t,1}}(x_j),\mu_{m_{t,2}}(x_j),\mu_{m_{t,3}}(x_j)\right\}\right.,t=1,2,\cdots,r,s'=1,2,3,\ j=1,2,\cdots,n\right\}. \quad (9)$$

Step 2.3: Respectively calculate the ratio of membership information entropy and distribution coefficient function. The ratio corresponds to each sample attribute of Step 2.2 as

$$V_{\eta_{t,s'}(x_j)} = E(\eta_{t,s'}(x_j)) / D(\eta_{t,s'}(x_j)). \quad (10)$$

Step 2.4: Let $\eta(x_j)$ be the simple fuzzy concept set, that is, $\eta(x_j) = \{\eta_{t,s'}(x_j) \mid t = 1, 2, \cdots, r; s' = 1,2,3; j = 1,2,\cdots,n\}$.

Step 2.5: Select the smallest value $\mu_a(x)$ which corresponds to attribute a; select the second smallest value $\mu_b(x)$ which corresponds to attribute b; $\mu_a(x_j)$ is defined as

$$\mu_a(x_j) = \inf_{\alpha \in \eta(x_j)} (L_\alpha(x_j)) \in [0,1]. \quad (11)$$

Let $\eta(x_j)' = \{\eta(x_j) - \{a\}\}$, and then $\mu_b(x_j)$ is defined as

$$\mu_b(x_j) = \inf_{\alpha \in \eta(x_j)'} (L_\alpha(x_j)) \in [0,1]. \quad (12)$$

Step 2.6: Apply Step 2.3 and find the evaluation index $V_{\eta_{t,s'}(x_j)}$ which corresponds to attribute a, and the evaluation index $V_{\eta_{t',s'}(x_j)}$ which corresponds to attribute b, and compare $V_{\eta_{t,s'}(x_j)}$ and $V_{\eta'_{t,s'}(x_j)}$.

Step 2.7: If $V_{\eta'_{t,s'}(x_j)} \geq V_{\eta_{t,s'}(x_j)}$, then eliminate attribute a, and the remaining sample attributes are expressed as $\eta(x_j)' = \{\eta(x_j) - \{a\}\}$, and return to Step 2.5, continue the recursion until $V_{\eta'_{t,s'}(x_j)} < V_{\eta_{t,s'}(x_j)}$.

Step 2.8: Return the final remaining attributes $\eta(x_j)'$ for each sample x_j, the remaining attributes can be used to describe the sample x_j, thereby, we get the fuzzy attributes of each sample via the above Steps 2.1–2.8.

Step 3: Clustering procedure based on the fuzzy attributes of each sample.

Step 3.1: The fuzzy description of each sample A_i is ζ_{A_i}, and we construct the fuzzy sample relation $r_{ij} = \min\left\{\mu_{\zeta_{A_i}\Lambda\zeta_{A_j}}(A_i), \mu_{\zeta_{A_i}\Lambda\zeta_{A_j}}(A_j)\right\}$. Based on universe of discourse $X = \{A_1, A_2, \cdots, A_n\}$, Liu (1998) demonstrated that the integer k exists to make $(\boldsymbol{M}_\Lambda^k)^2 = \boldsymbol{M}_\Lambda^k$, thereby, we can deduct the equivalence relation at universe of discourse X based on the fuzzy sample relation matrix $\boldsymbol{R} = \boldsymbol{M}_\Lambda^k = (r_{ij}^k)_{n\times n}$.

Step 3.2: Calculate the initial clustering results based on the fuzzy equivalence matrix.

Denote the diagonal element of fuzzy sample relation matrix as r_{ii}^k. It can be verified as $r_{ji}^k = r_{ij}^k \leq r_{ii}^k$, thereby we can identify the different membership function values based on the diagonal elements and other elements in the matrix. They are expressed as α_l $(l = 1,2,\cdots,g)$, and these values gradually increase according to the sequence of l. Find $r_{ij}^k = r_{ii}^k = \alpha_l$ in the fuzzy sample relation matrix; the corresponding samples can be grouped into one or multiple clusters, and the remaining samples where $r_{ij}^k > \alpha_l$ can be grouped as another cluster. This procedure will recursively continue until l=g.

Step 3.3: Calculate the weight of each fuzzy description within clusters.

The samples are divided into different clusters according to different α_l values, we suppose that the initial clusters are $p'_1, p'_2, \cdots, p'_{c'}$ based on α_l, and calculate the weight of each fuzzy description of p'_h: $\xi_{p'_h} = \{C_{p'_h}, w_{p'_h}\}$, $\xi_x = \{m_{g,s'} \mid g \in \{1,2,\cdots,r\}$, $s' = 1$ or 2 or $3, x \in p'_h\}$, ξ_x denotes the fuzzy attributes set of p'_h, $C_{p'_h} = \{m \mid m \in \xi_x, x \in p'_h\}$, $w_{p'_h} = \{w_m \mid m \in C_{p'_h}\}$, $w_m = \dfrac{|p_h'^m|}{\sum_{m \in C_{p'_h}} |p_h'^m|}$, $|p_h'^m| = |\{x \mid x \in p'_h, m \in \xi_x\}|$, where $|\cdot|$ is defined as the number of elements, and $w_m \in w_{p'_h}$ is the weight of $m \in C_{p'_h}$.

Step 3.4: Finalize the clusters based on the weighted membership function.

For each sample $x \in X$, calculate the weighted membership function values $f_{(c,w,h)}(x)=\sum_{m\in C_{p_h}} w_m L_m(x),\ \ h=1,2,\cdots,c'$, in each initial cluster, where $L_m(x)$ is the measure of initial sample, select $f_{(c,w,h^*)}(x)=\arg\max_{1\le h\le c'}\{f_{(c,w,h)}(x)\}$ which corresponds to h^*, then we can obtain $x\in p'_{h^*}$, and finally obtain the result of clusters P_1, P_2 ,…, P_c, and set $\xi_x=\{m_{g,s'}\mid g\in\{1,2,\cdots,r\}, s'=1,2,3, x\in P_h\}$, $C_{P_h}=\{m\mid m\in\xi_x, x\in P_h\}$.

Step 3.5: Calculate the clustering validity index.

According to different thresholds $\alpha\in[0,1]$, we can select the optimal result from the clustering validity index. Some notations are introduced to cluster the validity indices, N is the number of samples, $V=\{v_1,v_2,\cdots,v_c\}$ is the set of sample center in each cluster, $v_h=\left\{v_h^m=\sum_{x\in P_h}\rho_m(x)\Big/n_{P_h}\,\middle|\, m\in C_{P_h}\right\}, 1\le h\le c,$ where and n_{P_h} is the number of samples in cluster h. Thereby, the clustering validity index I_α can be expressed as follows:

$$I_\alpha=\frac{c\times(c-1)\sum_{h=1}^{c}\sum_{x\in P_h}\sum_{m\in M}\left\|\rho_m(x)-v_h^m\right\|^2}{2\times\alpha\times\left(\sum_{h=1}^{c}\sum_{k=1,h\ne k}^{c}\sum_{m\in M}\left\|v_h^m-v_k^m\right\|^2\right)},\tag{13}$$

where $RC_{hk}=\dfrac{\sum_{h=1}^{c}\sum_{k=1,h\ne k}^{c}\sum_{m\in M}\left\|v_h^m-v_k^m\right\|^2}{\dfrac{c\times(c-1)}{2}}$ describes the dispersion degree between clusters, which can be used for merging different clusters, and $\sum_{h=1}^{c}\sum_{x\in P_h}\sum_{m\in C_{P_h}}\left\|\rho_m(x)-v_h^m\right\|^2$ denotes the closeness of samples within each cluster. The clustering validity index I_α becomes smaller when the closeness within each cluster becomes smaller and the dispersion degree between clusters becomes greater. The cluster is the most distinct when I_α is the smallest value.

2.4 Ranking within clusters

The fuzzy TOPSIS approach (Awasthi *et al.*, 2011) is used to evaluate the potential locations based on the selected 16 sub-criteria. The goal is to find the positive ideal solution, where the cost criteria is minimized and the benefit criteria is maximized, and the negative ideal solution, where the benefit criteria is minimized and the cost criteria is maximized (Awasthi *et al.*, 2011; Li *et al.*, 2011). From the previous definitions, A'_h is the number of potential locations with cluster h, thereby, the fuzzy numbers of sub-criteria and potential locations are defined as: $W_{ut'}^2=(w_{ut'_1},w_{ut'_2},w_{ut'_3})$, $X_{uhit'}^2=(a_{uit'},b_{uit'},c_{uit'})$, $(u=1,2,\cdots,m;$ $h=1,2,\cdots,c;\ i=1,2,\cdots,n';\ t'=1,2,\cdots,k)$, respectively. The procedures of fuzzy TOPSIS approach are described as follows.

Step 1: Calculate the aggregate fuzzy ratings for the criteria and location alternatives.

Let $\tilde{w}_{t'}$ be the aggregated fuzzy weights for sub-criteria within each cluster, and $\tilde{w}_{t'}$ is expressed as $\tilde{w}_{t'}=(w_{t'_1},w_{t'_2},w_{t'_3})$, among which

$$\begin{aligned} w_{t'_1}&=\min_u\{w_{ut'_1}\},\\ w_{t'_2}&=\frac{1}{m}\sum_{u=1}^{m}w_{ut'_2},\\ w_{t'_3}&=\max_u\{w_{ut'_3}\}.\end{aligned}\tag{14}$$

Let $\tilde{x}_{t'_i}$ be the aggregated fuzzy rating of location alternatives within each cluster, and $\tilde{x}_{t'i}=(a_{it'},b_{it'},c_{it'})$ is expressed as

$$\begin{aligned} a_{it'}&=\min_u\{a_{uit'}\},\\ b_{it'}&=\frac{1}{m}\sum_{u=1}^{m}b_{uit'},\\ c_{it'}&=\max_u\{c_{uit'}\}.\end{aligned}\tag{15}$$

Step 2: Calculate the location decision matrix and fuzzy weight matrix.

Based on Step 1, the location decision matrix $\tilde{\boldsymbol{R}}$ and the fuzzy weight matrix $\tilde{\boldsymbol{W}}$ can be calculated as follows:

$$\tilde{\boldsymbol{R}}=\begin{bmatrix}\tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n'}\\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n'}\\ \cdots & \cdots & \cdots & \cdots\\ \tilde{x}_{k1} & \tilde{x}_{k2} & \cdots & \tilde{x}_{kn'}\end{bmatrix},\ \tilde{\boldsymbol{W}}=[\tilde{w}_1,\tilde{w}_2,\cdots,\tilde{w}_k].\quad(16)$$

Step 3: Calculate the positive ideal and negative ideal solutions.

Let $\tilde{\boldsymbol{R}}$ be the normalized fuzzy decision matrix, which is given as follows:

$$\tilde{\boldsymbol{R}}'=[\tilde{r}_{t'i}]_{k\times n'},\ t'=1,2,\cdots,k,\ i=1,2,\cdots,n',$$

for normalized cost criteria,

$$\tilde{r}_{t'i}=\left(\frac{a_{t'}^-}{c_{it'}},\frac{a_{t'}^-}{b_{it'}},\frac{a_{t'}^-}{a_{it'}}\right),\ a_{t'}^-=\min_i(a_{it'});\quad(17)$$

for normalized benefit criteria,

$$\tilde{r}_{t'i}=\left(\frac{a_{it'}}{c_{t'}^*},\frac{b_{it'}}{c_{t'}^*},\frac{c_{it'}}{c_{t'}^*}\right),\ c_{t'}^*=\max_i(c_{it'}).\quad(18)$$

Step 4: Calculate the weighted normalized matrix $\tilde{\boldsymbol{V}}$ by multiplying the fuzzy decision matrix $\tilde{\boldsymbol{R}}'$ with the fuzzy weight matrix $\tilde{\boldsymbol{W}}$, the $\tilde{v}_{t'i}$ is calculated as

$$\begin{gathered}\tilde{v}_{t'i}=\tilde{r}_{t'i}(.)\tilde{w}_{t'}=(v_{t'i_1},v_{t'i_2},v_{t'i_3}),\\ t'=1,2,\cdots,k,\ i=1,2,\cdots,n'.\end{gathered}\quad(19)$$

Step 5: Calculate the positive ideal and the negative ideal solutions,

$$C^*=\left(\tilde{v}_1^*,\tilde{v}_2^*,\cdots,\tilde{v}_k^*\right),$$

where

$$\tilde{v}_{t'}^*=\max_i(v_{t'i_3}),\ t'=1,2,\cdots,k,\ i=1,2,\cdots,n',\quad(20)$$

$$C^-=\left(\tilde{v}_1,\tilde{v}_2,\cdots,\tilde{v}_k\right),$$

where

$$\tilde{v}_{t'}^-=\min_i(v_{t'i_1}),\ t'=1,2,\cdots,k,\ i=1,2,\cdots,n'.\quad(21)$$

Step 6: Calculate the dimensional Euclidean distance.

The dimensional Euclidean distance of positive ideal solution is given as

$$d_i^*=\left\{\sum_{t'=1}^{k}(\tilde{v}_{t'i}-\tilde{v}_{t'}^*)^2\right\}^{1/2},\ i=1,2,\cdots,n'.\quad(22)$$

The dimensional Euclidean distance of negative ideal solution is given as

$$d_i^-=\left\{\sum_{t'=1}^{k}(\tilde{v}_{t'i}-\tilde{v}_{t'}^-)^2\right\}^{1/2},\ i=1,2,\cdots,n'.\quad(23)$$

Step 7: Calculate the closeness to the ideal solution.

Let CA_i be the ratio of the distance to the fuzzy negative ideal solution d_i^- and the sum of fuzzy positive ideal solution and fuzzy negative ideal solution, and it represents the relative closeness as follows:

$$\mathrm{CA}_i=\frac{d_i^-}{d_i^-+d_i^*},\ i=1,2,\cdots,n'.\quad(24)$$

The higher the value CA_i is, the more ideal the alternative is for the final location.

3 Implementation and comparisons

3.1 Data source

To illustrate the applicability of the proposed approach in multiple facilities location planning, an MDCL selection problem in Guiyang, China is used as a case study. Guiyang City is the capital of Guizhou Province and it is located in the north-central region of Guizhou Province, China. The logistics company owns a logistics center in Guiyang City, and the company needs to establish multiple distribution centers to extend the market. Eighteen alternative distribution centers have been chosen as shown in Fig. 2, which are expressed as A_1, A_2, …, A_{18}.

In order to accurately select the locations of distribution centers, three top decision-makers $D=\{D_1, D_2, D_3\}$ are invited to evaluate the criteria and alternative locations. The first and second are from the transportation planning department in Guiyang City, and the third is a senior manager who operates the logistics center. All three have many years of planning experience, and are very familiar with the alternative location surrounding conditions.

After interviewing the three top decision-makers, we obtained the weights for the 16 sub-criteria and the preferences for alternative locations, as shown in Table 2 and Table 3 (p.792−794).

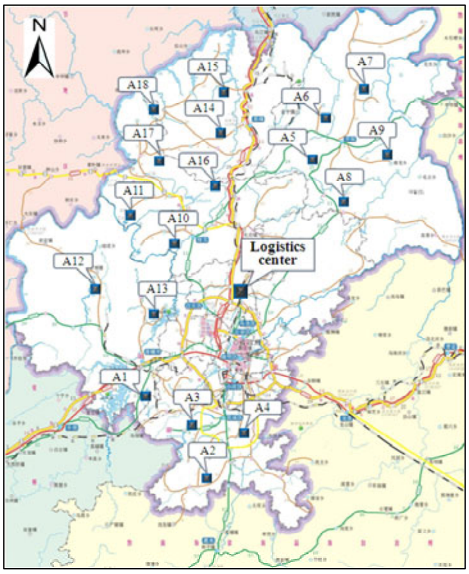

Fig. 2 Alternative distribution centers scatter diagram

Table 2 Linguistic assessments for the sub-criteria

Sub-criterion	Linguistic assessment		
	D_1	D_2	D_3
C_{11}	B.P&G	F	F
C_{12}	B.F&G	B.F&G	G
C_{13}	F	F	G
C_{21}	G	B.F&G	G
C_{22}	VG	B.G&VG	G
C_{23}	B.F&G	B.G&VG	G
C_{31}	B.G&VG	G	B.F&G
C_{32}	B.F&G	B.F&G	B.G&VG
C_{41}	B.P&F	B.P&F	B.F&G
C_{42}	P	F	B.P&F
C_{43}	G	B.F&G	G
C_{51}	G	G	B.F&G
C_{52}	B.G&VG	G	VG
C_{53}	B.F&G	F	F
C_{61}	B.F&G	B.P&F	F
C_{62}	F	F	B.P&F

3.2 Result analysis

According to Eqs. (6) and (7), we can obtain comprehensive evaluation values by integrating the sub-criteria into the major criteria. The evaluation values are presented in Table 4 (p.795).

Following Step 1–Step 3 in the clustering algorithm procedure, we can obtain the results as follows.

1. When threshold α=0.6932, $I_\alpha = 4.49$, there are two clusters:

$P_1=\{A_{10}, A_{11}, A_{12}, A_{13}\}$,

$P_2=\{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{14}, A_{15}, A_{16}, A_{17}, A_{18}\}$.

2. When threshold α=0.7119, $I_\alpha = 3.87$, there are three clusters:

$P_1=\{A_{10}, A_{11}, A_{12}, A_{13}\}$,

$P_2=\{A_1, A_2, A_3, A_4\}$,

$P_3=\{A_5, A_6, A_7, A_8, A_9, A_{14}, A_{15}, A_{16}, A_{17}, A_{18}\}$.

3. When threshold α=0.7256, $I_\alpha = 2.51$, there are four clusters:

$P_1=\{A_{10}, A_{11}, A_{12}, A_{13}\}$,

$P_2=\{A_1, A_2, A_3, A_4\}$,

$P_3=\{A_{14}, A_{15}, A_{16}, A_{17}, A_{18}\}$,

$P_4=\{A_5, A_6, A_7, A_8, A_9\}$.

4. When threshold α=0.7659, $I_\alpha = 3.03$, there are five clusters:

$P_1=\{A_{10}, A_{11}, A_{12}, A_{13}\}$,

$P_2=\{A_1, A_2, A_3, A_4\}$,

$P_3=\{A_{14}, A_{15}, A_{16}, A_{17}, A_{18}\}$,

$P_4=\{A_5, A_8, A_9\}$,

$P_5=\{A_6, A_7\}$.

Comparing all of the results, we can find that $I_{0.7256}$=2.51 is the smallest; thus, the option with four clusters is the best choice.

Then, we use the TOPSIS approach to rank the alternative locations within each cluster. According to Eqs. (14)−(24), the final ranking results are shown in Table 5 (p.795) as follows.

By comparing the CA_i values in each cluster (Table 5), A_{13}, A_4, A_{14}, and A_8 are selected as the final urban distribution center locations for the logistics company. According to Eq. (13), we can calculate the dispersion degree between clusters. The ranking results of dispersion degree between clusters is $\text{RC}_{34} > \text{RC}_{13} > \text{RC}_{12} > \text{RC}_{24}$; the higher the RC_{hk} between clusters is, the smaller the closeness between clusters is. Based on the demand for the economic development in each region, the decision-makers can also merge possible clusters, and finalize the number of distribution centers in each cluster.

Table 3a Linguistic assessments for alternatives by D_1

Sub-criteria	Linguistic assessments for alternatives by D_1								
	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
C_{11}	L	M	L	B.VL&L	M	M	M	B.L&M	B.L&M
C_{12}	B.L&M	L	B.L&M	L	M	B.L&M	B.L&M	M	M
C_{13}	B.VL&L	VL	L	B.L&M	B.L&M	M	B.M&H	M	B.L&M
C_{21}	B.H&VH	B.H&VH	H	H	B.M&H	H	H	M	H
C_{22}	VH	H	VH	B.H&VH	H	B.M&H	H	H	H
C_{23}	B.M&H	B.M&H	H	B.H&VH	M	B.M&H	M	B.M&H	M
C_{31}	B.M&H	H	B.M&H	H	B.H&VH	H	B.H&VH	VH	H
C_{32}	H	B.M&H	H	B.M&H	B.H&VH	VH	B.H&VH	H	VH
C_{41}	M	B.M&H	M	B.M&H	B.L&M	B.VL&L	B.VL&L	B.L&M	B.L&M
C_{42}	B.L&M	B.L&M	B.L&M	B.L&M	B.L&M	L	L	L	B.L&M
C_{43}	B.M&H	M	B.M&H	M	L	B.L&M	B.L&M	B.VL&L	L
C_{51}	B.H&VH	H	B.H&VH	VH	M	H	B.M&H	B.M&H	H
C_{52}	B.H&VH	VH	VH	B.H&VH	B.H&VH	B.M&H	H	H	H
C_{53}	VH	H	B.H&VH	B.H&VH	B.M&H	H	B.M&H	H	B.M&H
C_{61}	B.M&H	M	B.M&H	B.M&H	M	L	B.VL&L	B.L&M	B.L&M
C_{62}	M	B.M&H	M	M	B.VL&L	B.L&M	M	B.VL&L	B.L&M
Sub-criteria	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	A_{16}	A_{17}	A_{18}
C_{11}	B.H&VH	H	B.H&VH	B.H&VH	B.M&H	M	B.M&H	B.H&VH	B.M&H
C_{12}	VH	VH	B.H&VH	VH	M	M	M	M	B.M&H
C_{13}	VH	VH	VH	B.H&VH	B.M&H	H	B.M&H	B.L&M	M
C_{21}	VL	VL	B.VL&L	VL	B.VL&L	L	L	B.VL&L	L
C_{22}	VL	B.VL&L	VL	B.VL&L	B.VL&L	B.VL&L	L	B.VL&L	B.VL&L
C_{23}	B.VL&L	VL	VL	VL	L	L	B.VL&L	B.L&M	L
C_{31}	B.L&M	L	B.L&M	B.VL&L	VL	B.VL&L	VL	B.VL&L	VL
C_{32}	B.VL&L	L	B.VL&L	B.L&M	B.VL&L	VL	B.VL&L	VL	B.VL&L
C_{41}	B.M&H	B.M&H	B.H&VH	H	VH	H	B.H&VH	B.H&VH	VH
C_{42}	B.H&VH	B.M&H	B.H&VH	B.H&VH	B.H&VH	B.H&VH	VH	H	B.H&VH
C_{43}	M	H	B.M&H	B.M&H	B.H&VH	VH	H	VH	B.H&VH
C_{51}	B.VL&L	L	VL	L	B.L&M	L	L	B.L&M	B.L&M
C_{52}	VL	VL	L	VL	B.VL&L	B.L&M	B.VL&L	L	B.VL&L
C_{53}	B.VL&L	B.VL&L	L	B.VL&L	L	B.VL&L	M	B.VL&L	L
C_{61}	B.M&H	H	B.H&VH	H	B.H&VH	VH	B.H&VH	VH	B.H&VH
C_{62}	H	H	H	B.H&VH	VH	B.H&VH	VH	B.H&VH	VH

Table 3b Linguistic assessments for alternatives by D_2

Sub-criteria	Linguistic assessments for alternatives by D_2								
	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
C_{11}	M	B.L&M	L	B.VL&L	L	L	B.L&M	M	M
C_{12}	B.VL&L	L	B.VL&L	L	B.L&M	B.M&H	B.L&M	B.L&M	B.L&M
C_{13}	B.VL&L	B.VL&L	B.L&M	B.L&M	M	L	B.M&H	B.M&H	M
C_{21}	B.H&VH	H	H	B.M&H	H	H	B.H&VH	M	B.M&H
C_{22}	VH	B.M&H	H	H	B.M&H	M	B.M&H	B.M&H	H
C_{23}	M	B.H&VH	VH	VH	B.M&H	H	M	H	M
C_{31}	B.L&M	M	B.M&H	H	B.H&VH	V.H&VH	H	H	VH
C_{32}	VH	B.H&VH	H	B.M&H	B.H&VH	B.H&VH	B.H&VH	VH	H
C_{41}	B.M&H	L	H	B.L&M	B.L&M	B.VL&L	B.VL&L	L	L
C_{42}	L	B.L&M	M	B.M&H	B.L&M	L	L	B.L&M	M
C_{43}	B.M&H	H	B.L&M	M	L	B.L&M	B.L&M	B.VL&L	L
C_{51}	H	B.H&VH	H	VH	M	B.M&H	H	B.M&H	B.M&H
C_{52}	VH	VH	VH	B.H&VH	B.H&VH	H	B.M&H	H	H
C_{53}	VH	B.H&VH	H	B.H&VH	B.M&H	H	H	H	B.M&H
C_{61}	M	H	B.M&H	M	B.VL&L	M	M	M	L
C_{62}	B.M&H	L	M	B.M&H	M	VL	B.L&M	B.VL&L	M
Sub-criteria	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	A_{16}	A_{17}	A_{18}
C_{11}	VH	B.H&VH	H	B.H&VH	B.M&H	B.M&H	M	B.M&H	B.M&H
C_{12}	B.H&VH	VH	VH	VH	B.M&H	M	B.H&VH	M	M
C_{13}	VH	B.H&VH	H	B.H&VH	M	B.M&H	B.L&M	B.M&H	B.M&H
C_{21}	B.VL&L	VL	VL	B.VL&L	L	L	B.VL&L	L	B.VL&L
C_{22}	B.VL&L	VL	B.VL&L	VL	B.VL&L	B.L&M	B.VL&L	B.VL&L	B.VL&L
C_{23}	VL	B.VL&L	VL	VL	L	VL	B.L&M	L	L
C_{31}	L	B.L&M	B.VL&L	B.L&M	B.VL&L	VL	L	VL	VL
C_{32}	B.L&M	B.VL&L	B.L&M	B.VL&L	VL	B.VL&L	VL	L	B.VL&L
C_{41}	B.H&VH	B.M&H	M	B.H&VH	B.H&VH	H	B.H&VH	VH	VH
C_{42}	VH	B.M&H	H	B.H&VH	VH	B.H&VH	H	B.H&VH	B.H&VH
C_{43}	M	H	B.M&H	B.M&H	H	VH	VH	B.H&VH	B.H&VH
C_{51}	L	B.VL&L	VL	B.VL&L	B.L&M	L	B.L&M	L	L
C_{52}	VL	L	B.VL&L	VL	B.VL&L	B.L&M	B.VL&L	B.L&M	B.VL&L
C_{53}	B.VL&L	VL	L	B.VL&L	L	B.VL&L	L	B.VL&L	,M
C_{61}	B.H&VH	B.M&H	H	B.M&H	H	B.H&VH	VH	B.H&VH	VH
C_{62}	B.M&H	VH	B.H&VH	H	VH	VH	B.H&VH	VH	B.H&VH

Table 3c Linguistic assessments for alternatives by D_3

Sub-criteria	Linguistic assessments for alternatives by D_3								
	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
C_{11}	B.L&M	L	B.VL&L	B.L&M	M	B.L&M	M	M	M
C_{12}	L	B.L&M	L	L	B.L&M	M	B.L&M	B.L&M	B.L&M
C_{13}	L	B.VL&L	B.L&M	B.VL&L	B.L&M	B.L&M	M	M	B.M&H
C_{21}	VH	H	VH	B.H&VH	B.H&VH	M	H	B.M&H	H
C_{22}	B.H&VH	B.M&H	H	H	B.M&H	B.M&H	M	H	B.M&H
C_{23}	B.M&H	B.H&VH	H	B.H&VH	M	H	H	M	B.M&H
C_{31}	M	B.M&H	H	H	H	H	VH	B.H&VH	B.H&VH
C_{32}	B.H&VH	H	B.M&H	B.M&H	B.H&VH	VH	H	B.H&VH	B.H&VH
C_{41}	B.L&M	B.L&M	B.M&H	H	L	L	B.L&M	B.VL&L	B.VL&L
C_{42}	L	B.M&H	L	M	B.L&M	M	B.L&M	L	L
C_{43}	H	M	B.M&H	B.L&M	B.VL&L	L	L	B.L&M	B.L&M
C_{51}	H	B.H&VH	VH	B.H&VH	B.M&H	B.M&H	M	H	H
C_{52}	VH	B.H&VH	B.H&VH	VH	H	H	B.H&VH	H	B.M&H
C_{53}	H	VH	B.H&VH	B.H&VH	H	B.M&H	B.M&H	B.M&H	H
C_{61}	H	B.M&H	M	L	M	L	B.VL&L	M	M
C_{62}	L	M	B.M&H	H	B.VL&L	M	M	VL	B.L&M
Sub-criteria	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	A_{16}	A_{17}	A_{18}
C_{11}	B.H&VH	B.H&VH	VH	H	M	B.M&H	B.M&H	B.M&H	B.M&H
C_{12}	VH	VH	B.H&VH	VH	B.H&VH	B.M&H	M	M	M
C_{13}	B.H&VH	B.H&VH	VH	VH	B.L&M	M	B.M&H	B.M&H	B.M&H
C_{21}	VL	B.VL&L	B.VL&L	VL	B.VL&L	L	B.VL&L	L	L
C_{22}	B.VL&L	VL	B.VL&L	VL	B.VL&L	B.VL&L	B.VL&L	L	B.VL&L
C_{23}	VL	VL	VL	B.VL&L	B.L&M	L	L	B.VL&L	L
C_{31}	B.VL&L	B.L&M	L	B.L&M	VL	VL	B.VL&L	VL	L
C_{32}	B.L&M	B.VL&L	B.L&M	B.VL&L	L	B.VL&L	VL	B.VL&L	VL
C_{41}	M	B.H&VH	B.M&H	B.H&VH	B.H&VH	VH	VH	B.H&VH	H
C_{42}	H	B.H&VH	B.M&H	VH	H	B.H&VH	B.H&VH	VH	B.H&VH
C_{43}	B.M&H	B.M&H	H	M	VH	B.H&VH	B.H&VH	H	VH
C_{51}	VL	B.VL&L	L	L	L	B.L&M	B.L&M	L	B.L&M
C_{52}	B.VL&L	VL	VL	B.VL&L	B.VL&L	B.VL&L	B.VL&L	B.L&M	B.VL&L
C_{53}	L	B.VL&L	B.VL&L	VL	M	L	L	B.VL&L	L
C_{61}	VH	H	B.M&H	B.H&VH	VH	H	VH	VH	B.H&VH
C_{62}	B.M&H	B.H&VH	H	B.M&H	B.H&VH	VH	B.H&VH	B.H&VH	VH

3.3 Algorithm comparisons

Based on the fuzzy set theory, Chou *et al.* (2008) presented a new fuzzy multiple attributes decision-making approach considering the heterogeneous impact of each decision-maker. They applied the factor rating system and simple additive weighting approach to evaluate facility location alternatives. Li *et al.* (2011) presented another comprehensive methodology for the logistics center location selection. For comparison purposes, we implemented Chou *et al.* (2008)'s algorithm and Li *et al.* (2011)'s algorithm into our case study in the same context with four distribution centers. The algorithm proposed by Chou *et al.* (2008) generates the final locations as A_4, A_{10}, A_{11}, and A_{13}, and Li *et al.* (2011)'s algorithm results in the final locations as A_{10}, A_{13}, A_{14}, and A_{16}. They are shown in Fig. 3 as follows.

The proposed algorithm provides a novel approach to address the MDCL problem. The final distribution centers calculated by Chou *et al.* (2008)'s approach and Li *et al.* (2011)'s approach are located along one side of the logistics center in our case study, which may incur more traffic congestion events near the distribution centers, and increase transportation costs for customers living far away from these distribution centers. The potential distribution centers by our proposed approach, however, are scattered more evenly around the logistics center, which adheres to reality more agreeably. Each attribute of the alternative locations in our proposed approach is further split into several finer attributes during clustering, which gains the heterogeneity between alternative locations, but the algorithms developed by Li *et al.* (2011) and Chou *et al.* (2008) can not take into account these factors.

Due to the imparity of natural condition, business environment and environmental impact among those distribution center locations, both Chou *et al.* (2008)'s approach and Li *et al.* (2011)'s approach are not designed to incorporate these factors. In our proposed approach, the clustering algorithm is first used to group these locations with similar attributes into one category, and the location selection

Table 4 Aggregate evaluation matrix for alternatives

Criterion	Aggregate evaluation for alternative					
	A_1	A_2	A_3	A_4	A_5	A_6
C_1	0.5788	0.5727	0.5709	0.5721	0.6270	0.6234
C_2	0.8227	0.7968	0.8257	0.8198	0.7654	0.7634
C_3	0.7598	0.7616	0.7639	0.7653	0.8167	0.8230
C_4	0.6123	0.6098	0.6179	0.6159	0.5526	0.5450
C_5	0.8293	0.8293	0.8295	0.8379	0.7609	0.7665
C_6	0.6547	0.6547	0.6615	0.6528	0.5844	0.5836
Criterion	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}
C_1	0.6424	0.6409	0.6378	0.7864	0.7781	0.7739
C_2	0.7680	0.7556	0.7659	0.5678	0.5638	0.5680
C_3	0.8178	0.8244	0.8244	0.6248	0.6173	0.6248
C_4	0.5477	0.5373	0.5519	0.6632	0.6736	0.6680
C_5	0.7615	0.7711	0.7667	0.5713	0.5718	0.5709
C_6	0.5962	0.5758	0.6106	0.7212	0.7316	0.7259
Criterion	A_{13}	A_{14}	A_{15}	A_{16}	A_{17}	A_{18}
C_1	0.7781	0.6767	0.6666	0.6857	0.6646	0.6763
C_2	0.5638	0.6055	0.6105	0.6060	0.6095	0.6030
C_3	0.6209	0.5641	0.5591	0.5651	0.5641	0.5646
C_4	0.6878	0.7242	0.7256	0.7242	0.7242	0.7286
C_5	0.5716	0.6182	0.6166	0.6182	0.6169	0.6182
C_6	0.7200	0.7691	0.7691	0.7757	0.7757	0.7753

Table 5 Ranking order for alternative distribution centers

Index	Class P_1				Class P_3				
	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	A_{16}	A_{17}	A_{18}
d^*	2.978	3.011	3.105	2.942	3.160	3.285	3.216	3.225	3.208
d^-	3.021	2.886	2.912	3.015	2.893	2.755	2.845	2.786	2.736
CA_i	0.503	0.489	0.484	0.506	0.478	0.456	0.469	0.463	0.460
Ranking order	$CA_{13}>CA_{10}>CA_{11}>CA_{12}$				$CA_{14}>CA_{16}>CA_{17}>CA_{18}>CA_{15}$				
Index	Class P_2				Class P_4				
	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
d^*	3.296	3.353	3.234	3.121	3.143	3.301	3.269	3.135	3.232
d^-	2.905	2.899	2.972	3.075	2.931	2.915	2.878	2.976	2.883
CA_i	0.468	0.464	0.479	0.496	0.483	0.469	0.468	0.487	0.471
Ranking order	$CA_4>CA_3>CA_1>CA_2$				$CA_8>CA_5>CA_9>CA_6>CA_7$				

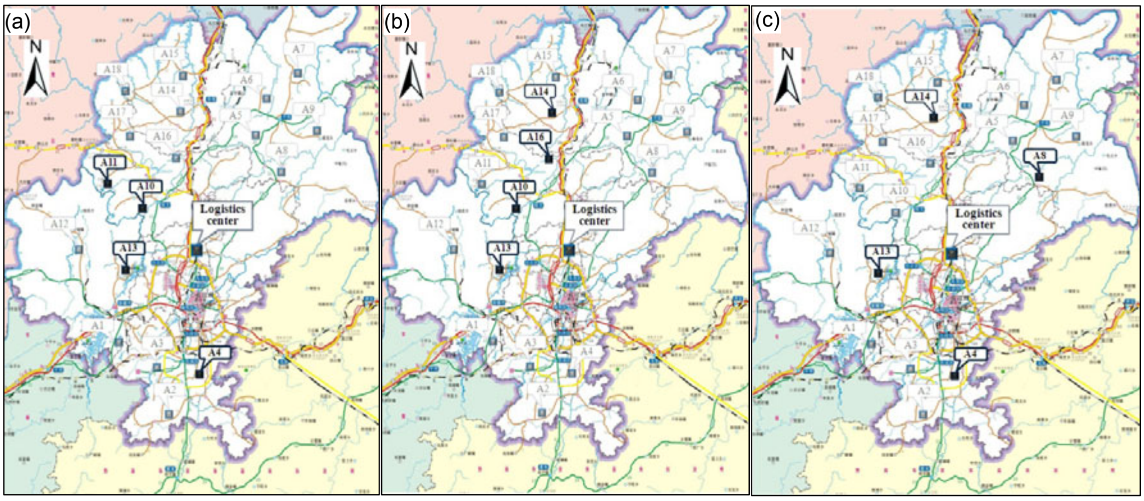

Fig. 3 Results of three algorithms
(a) Chou *et al.* (2008)'s algorithm; (b) Li *et al.* (2011)'s algorithm; (c) Proposed algorithm

procedure is then performed within each category. Compared with Chou *et al.* (2008) and Li *et al.* (2011)'s approaches, the proposed approach is able to handle the scenario that multiple heterogeneous regions share the same criterion for the distribution center location selection. In summary, the proposed algorithm should be more suitable for MDCL problems in real life.

3.4 Sensitivity analysis

To further investigate the impacts of different criteria weights on the location selection procedure, a sensitivity analysis is also conducted. A total of 27 experiments were undertaken in each cluster. The sub-criteria C_{43}, C_{61} and C_{62} are all cost criteria, while the remaining sub-criteria are all benefit criteria. In each cluster experiments 1–9 have the weights of all criteria set equally to VL, B.VL&L, L, B.L&M, M, B.M&H, H, B.H&VH, VH; experiments 10–25 have the weight of one criterion set at the highest weight VH, with the remaining set at the lowest weight VL; experiment 26 have the cost criteria (C_{43}, C_{61}, C_{62}) set at the highest weight VH, with the remaining criteria set at the lowest weight VL; and in experiment 27, the weights of the cost criteria are set at the lowest weight VL, with the remaining criteria set at the highest weight VH. The results of the analysis can be observed in Figs. 4a–4d.

Results indicate that for cluster P_1, in the experiments 10, 20, 25 and 26, the final location has been changed to A_{10} due to $\mathrm{CA}_{10} > \mathrm{CA}_{13}$; for cluster P_2, in the experiments 12, 20, 25 and 26, the location has been changed to A_3 due to $\mathrm{CA}_3 > \mathrm{CA}_4$; for cluster P_3, in the experiments 14, 16, 20, 24, 25 and 26, the location has been changed to A_{16} due to $\mathrm{CA}_{16} > \mathrm{CA}_{14}$; for cluster P_4, in the experiments 13, 19, 20, 24, 25 and 26, the location has been changed to A_5 due to $\mathrm{CA}_5 > \mathrm{CA}_8$. Through these experiments we can say that the location decision is relatively insensitive to benefit criteria weights; however, they are sensitive to cost criteria weights. From Figs. 4a–4d, we also observe that the cost criteria weights in clusters P_3 and P_4 are more sensitive than clusters P_1 and P_2, which implies that decision-makers need to assess the cost criteria weights more seriously than the benefit criteria for location planning.

4 Conclusions

This paper develops an innovative approach for selecting MDCLs under fuzzy environments. The hierarchical criteria analysis structure is initially constructed for planning distribution center locations. A fuzzy set approach is designed to convert the linguistic criteria ratings and alternative ratings into fuzzy numbers, and several relative definitions are also presented for algorithm procedures. Fuzzy integration algorithm is then proposed to synthesizing multiple lower hierarchical criteria into a higher hierarchical criterion, followed by an improved AFS approach

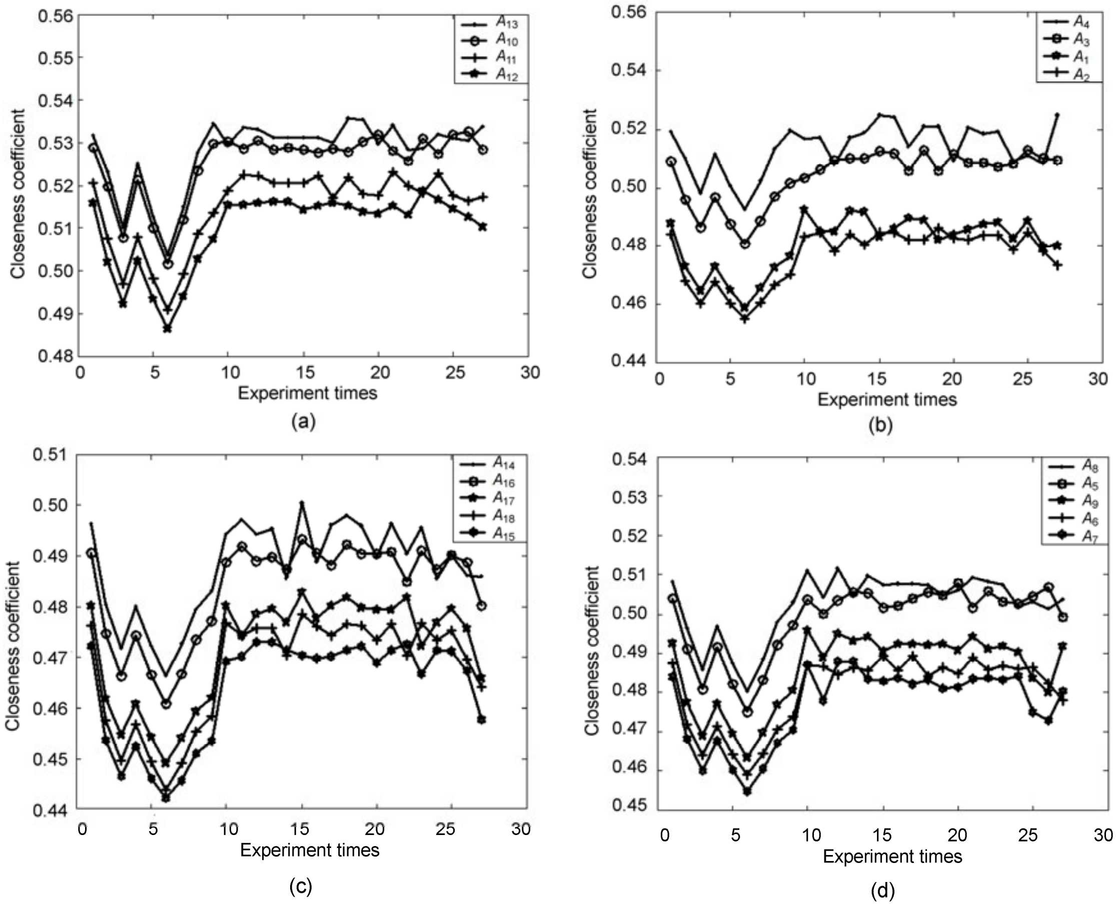

Fig. 4 Results of sensitivity analysis in cluster P_1 (a), cluster P_2 (b), cluster P_3 (c) and cluster P_4 (d)

based fuzzy clustering algorithm to group similar alternative locations; furthermore, a fuzzy TOPSIS approach is integrated to seek the optimal distribution center locations within clusters.

The proposed method has been successfully applied to assisting decision-makers in selecting distribution centers for a logistics company in Guiyang City, China. With further comparisons with other location selection algorithms, our approach has been proven to be more effective for MDCL problems. A sensitivity analysis is conducted in capturing the impacts of different evaluation criteria weights. The results demonstrate our approach performs very well as a cost-effective decision-support tool for distribution center location planning strategies. Additionally, the approach can also be straightforwardly extended to solving other similar multiple facility locations problem for decision-makers.

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