Journal of Zhejiang University SCIENCE
ISSN 1009-3095
http://www.zju.edu.cn/jzus
E-mail: jzus@zju.edu.cn

Solution to 1-D consolidation of non-homogeneous soft clay*

XIE Kang-he (谢康和)[†1], WEN Jie-bang (温介邦)[†1], XIA Jian-zhong (夏建中)[2]

([1]*Institute of Geotechnical Engineering, Zhejiang University, Hangzhou 310027, China*)

([2]*Department of Civil Engineering, Zhejiang University of Science and Technology, Hangzhou 310012, China*)

[†]E-mail: zdkhxie@zju.edu.cn; 97crazyiori@163.com

Received Jan. 10, 2005; revision accepted July 17, 2005

Abstract: In this work, semi-analytical methods were used to solve the problem of 1-D consolidation of non-homogeneous soft clay with spatially varying coefficients of permeability and compressibility. The semi-analytical solution was programmed and then verified by comparison with the obtained analytical solution of a special case. Based on the results of some computations and comparisons with the 1-D homogeneous consolidation (by Terzaghi) and the 1-D non-linear consolidation theory (by Davis *et al.*) of soft clay, some diagrams were prepared and the relevant consolidation behavior of non-homogeneous soils is discussed. It was shown that the result obtained differs greatly from Terzaghi's theory and that of the non-linear consolidation theory when the coefficients of permeability and compressibility vary greatly.

Key words: Non-homogeneous soil, Time-dependent loading, One-dimensional consolidation, Layered system, Semi-analytical solution

doi:10.1631/jzus.AS0029 **Document code:** A **CLC number:** TU432

INTRODUCTION

It is well known that taking the coefficient of consolidation c_v as a constant is a major shortcoming of conventional consolidation theory (Duncan, 1993). To overcome this shortcoming, many studies were conducted on the 1-D nonlinear consolidation problem, in which the coefficient of permeability k_v and that of volume compressibility m_v both directly relate to c_v and vary with the effective stress and void ratio (Davis and Raymond, 1965; Xie and Leo, 1999; Li *et al.*, 1999a; 1999b; Xie *et al.*, 2002). Meanwhile, the problem of 1-D consolidation of non-homogeneous soil was studied assuming that k_v and m_v vary with depth. Schiffman and Gibson (1964) first studied in detail the 1-D consolidation problem of non-homogeneous soil. By assuming k_v and m_v to be polynomial function or exponential function of depth and adopting differential method, they obtained a solution to 1-D consolidation of single layer soft clay under instantaneous loading.

Jiang *et al.*(2005) presented an analytical solution to the problem of 1-D consolidation of single-layered soil, wherein k_v is constant and E_s varies with depth, with this problem considered as a special case of the problem of consolidation of non-homogeneous soil above. The semi-analytical solution for the consolidation problem of single-layered or multi-layered soil with k_v and m_v (=1/E_s) varying with depth, but this solution may only be applicable to NC clay. Based on this solution, the consolidation behavior can be discussed. For either single-layered foundation or multi-layered foundation, this paper takes into consideration both single-drainage or double-drainage situations and time-dependent loading.

MATHEMATICAL MODELING AND ANALYTICAL SOLUTIONS OF SPECIAL CASE

The analysis scheme of consolidation problem of

*Project (No. 20030335027) supported by the National Research Foundation for the Doctoral Program of Higher Education of China

soft clay is shown in Fig.1 based on the assumption that k_{v} and m_{v} vary with depth. In this figure, H is the total thickness of soil layer. $q(t)$ is the uniformly distributed load applied on the top surface of the soil as shown in Fig.2, in which, q_0, q_{u} are the initial and ultimate load respectively, and t_{c} is the construction time. The top surface of the soil layer is pervious but the bottom is pervious or impervious.

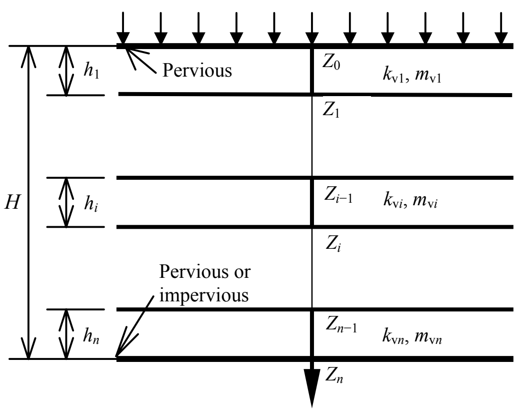

Fig.1 Loading and boundary condition of non-homogeneous foundation

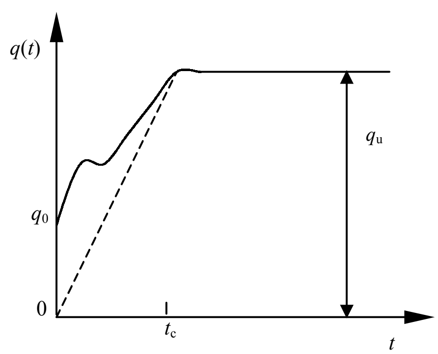

Fig.2 Loading-time relationship

Schiffman and Gibson (1964)'s equations governing 1-D consolidation process of single-layered non-homogeneous soft clay were used to obtain the following governing equations for the 1-D consolidation of layered non-homogeneous soil:

$$\frac{\partial^2 u_i}{\partial z^2}+\frac{1}{k_{\mathrm{v}i}(z)}\frac{\mathrm{d}k_{\mathrm{v}i}}{\mathrm{d}z}\frac{\partial u_i}{\partial z}=\frac{1}{c_{\mathrm{v}i}(z)}\left(\frac{\partial u_i}{\partial t}-\frac{\mathrm{d}q(t)}{\mathrm{d}t}\right),\quad (i=1,2,\ldots,n). \tag{1}$$

The solution conditions corresponding to Eq.(1) can be written as follows:

$t=0$: $u_i=q(0)=q_0$,
$z=0$: $u_1=0$,
$z=H$: $\partial u_n/\partial z=0$ (for single-drainage situation),
$u_n=0$ (for double-drainage situation),
$z=z_i$: $u_i=u_{i+1}$, $k_{\mathrm{v}i}\partial u_i/\partial z=k_{\mathrm{v}(i+1)}\partial u_{i+1}/\partial z$,
$i=1,2\ldots,n-1$

in which $k_{\mathrm{v}i}=k_{\mathrm{v}i}(z)$, $m_{\mathrm{v}i}=m_{\mathrm{v}i}(z)=1/E_{\mathrm{s}i}(z)$, $c_{\mathrm{v}i}=k_{\mathrm{v}i}/(\gamma_{\mathrm{w}}m_{\mathrm{v}i})=k_{\mathrm{v}i}E_{\mathrm{s}i}/\gamma_{\mathrm{w}}$, the coefficient of permeability, the coefficient of volume compressibility and the coefficient of consolidation in layer i respectively; γ_{w} is the unit weight of water; u_i is the excess porewater pressure in layer i.

The solution (Jiang *et al*., 2005) to the 1-D consolidation problem of single-layered non-homogeneous soil can be obtained as follows by assuming that the coefficient of permeability k_{v} is constant that the modulus of compressibility E_{s} varies linearly with depth, and that this solution may only be applicable to NC clay.

$$u=\sum_{m=1}^{\infty}\left\{\frac{2}{\lambda_m}\frac{D_m}{E_m}x[J_1(\lambda_m a)Y_1(\lambda_m x)-Y_1(\lambda_m a)J_1(\lambda_m x)]\times\mathrm{e}^{-\lambda_m^2 T_{\mathrm{v}}}\left(q_0+\int_0^t\frac{\mathrm{d}q}{\mathrm{d}\tau}\mathrm{e}^{\lambda_m^2 c_{\mathrm{v}0}/H^2\tau}\mathrm{d}\tau\right)\right\}, \tag{2}$$

$$U_{\mathrm{s}}=\frac{q(t)}{q_{\mathrm{u}}}-\frac{4}{q_{\mathrm{u}}\ln(1+\alpha)}\times\sum_{m=1}^{\infty}\left[\frac{1}{\lambda_m^2}\frac{D_m^2}{E_m}\mathrm{e}^{-\lambda_m^2 T_{\mathrm{v}}}\left(q_0+\int_0^t\frac{\mathrm{d}q}{\mathrm{d}\tau}\mathrm{e}^{\lambda_m^2 c_{\mathrm{v}0}/H^2\tau}\mathrm{d}\tau\right)\right], \tag{3}$$

where U_{s} is the average consolidation degree defined in terms of settlement or strain.

$$U_{\mathrm{p}}=\frac{q(t)}{q_{\mathrm{u}}}-\frac{\alpha}{q_{\mathrm{u}}}\sum_{m=1}^{\infty}\left[\frac{1}{\lambda_m^2}\frac{D_m F_m}{E_m}\mathrm{e}^{-\lambda_m^2 T_{\mathrm{v}}}\times\left(q_0+\int_0^t\frac{\mathrm{d}q}{\mathrm{d}\tau}\mathrm{e}^{\lambda_m^2 c_{\mathrm{v}0}/H^2\tau}\mathrm{d}\tau\right)\right], \tag{4}$$

where U_{p} is the average consolidation degree defined in terms of porewater pressure or stress; $a=2/\alpha$, $b=2\sqrt{1+\alpha}/\alpha$, $x=2\sqrt{1+\alpha z/H}/\alpha$,

$$D_m=J_1(\lambda_m a)[Y_0(\lambda_m a)-Y_0(\lambda_m b)]-Y_1(\lambda_m a)[J_0(\lambda_m a)-J_0(\lambda_m b)]. \tag{5}$$

$$E_m = b^2[J_1(\lambda_m b)Y_1(\lambda_m a) - Y_1(\lambda_m b)J_1(\lambda_m a)]^2 - a^2[J_0(\lambda_m a)Y_1(\lambda_m a) - Y_0(\lambda_m a)J_1(\lambda_m a)]^2,$$

(for single-drainage situation) (6.1)

$$E_m = b^2[J_0(\lambda_m b)Y_1(\lambda_m a) - Y_0(\lambda_m b)J_1(\lambda_m a)]^2 - a^2[J_0(\lambda_m a)Y_1(\lambda_m a) - Y_0(\lambda_m a)J_1(\lambda_m a)]^2.$$

(for double-drainage situation) (6.2)

$$F_m = J_1(\lambda_m a)[b^2 Y_2(\lambda_m b) - a^2 Y_2(\lambda_m a)] - Y_1(\lambda_m a)[b^2 J_2(\lambda_m b) - a^2 J_2(\lambda_m a)]. \quad (7)$$

The positive eigenvalues λ_m satisfy the following eigen-equation:

$$J_1(\lambda_m a)Y_0(\lambda_m b) - Y_1(\lambda_m a)J_0(\lambda_m b) = 0,$$

(for single-drainage situation) (8.1)

$$J_1(\lambda_m a)Y_1(\lambda_m b) - Y_1(\lambda_m a)J_1(\lambda_m b) = 0.$$

(for double-drainage situation) (8.2)

SEMI-ANALYTICAL SOLUTION UNDER NORMAL CONDITION

Basic idea

For general 1-D consolidation problem of single-layered or multi-layered non-homogeneous soil (Fig.1) under time-dependent loading (Fig.2), there seems to be no analytical solutions developed so far. Therefore, this paper adopts semi-analytical methods for solving the problem of 1-D consolidation of non-homogeneous soft clay. For some problems that have been studied thoroughly in theory, the numerical method can be substituted by corresponding semi-analytical method. Thus, the quantity of work can be greatly decreased.

In the paper, by dividing the whole clay layer into many small clay layers until the coefficient of permeability k_v and the coefficient of volume compressibility m_v can be considered as constant in each small clay layer, the semi-analytical method is used to convert the consolidation problem of non-homogeneous clay of the whole clay stratum to the linear consolidation problem of many small clay layer. Thus, the analytical solution (Xie *et al.*, 1995) for 1-D linear consolidation problem of layered soil under time-dependent loading can be used for calculation.

This paper developed the computation program to calculate in two cases below: (1) k_{vi} and m_{vi} of each clay layer are the function of depth (z), namely, $m_{vi} = m_{v0i}[1+\alpha_i(z-h_{0,i-1})/h_i]^{p_i}$, $k_{vi} = k_{v0i}[1+\beta_i(z-h_{0,i-1})/h_i]^{q_i}$, in which, m_{v0i}, k_{v0i} are the coefficient of volume compressibility and the coefficient of permeability in the top of layer i respectively; $h_{0,i-1}$ is the total thickness from layer 1 to layer $i-1$; h_i is the thickness of layer i; $\alpha_i, \beta_i, p_i, q_i$ are parameters that can be determined according to the properties of each clay layer. When $i=1, p_1=1, \beta_1=0$, the case becomes the one discussed in (Jiang *et al.*, 2005). (2) For soft clay, k_v and m_v are supposed to distribute not-uniformly because the initial effective pressure varies with depth.

$$m_{vi} = m_{v0i} = \frac{0.434 C_{ci}}{(1+e_{0i})\sigma'_{0i}}, \quad k_{vi} = k_{v0i} = k_{v1i}(\sigma'_{1i}/\sigma'_{0i})^{C_{ci}/C_{ki}}, \quad (9)$$

where e_{0i}, σ'_{0i} are the initial void ratio and the initial effective pressure; C_{ci}, C_{ki} are the compression index and the permeability index; k_{v1i}, σ'_{1i} are the coefficients of permeability and the effective pressure corresponding to void ratio e_{1i} at any point "1" on the e_i−log σ'_{0i} curve and e−logk_{vi} curve of the layer i respectively.

Validation and virtues of semi-analytical method

The semi-analytical approach was validated by comparing the semi-analytical solution in this paper against the analytical solution for single-layered non-homogeneous soil with linearly varying modulus of compressibility. The average degree of consolidation U_s defined in terms of settlement under instantaneous loading and the average degree of consolidation U_p defined in terms of the average excess pore in Fig.3 and Fig.4 respectively, in which the parameter α=−0.9, 0.1, 1, 9 was taken into calculation. It can be seen that the semi-analytical solution agrees quite well with the analytical solution. In addition, two methods' calculated results for the excess porewater pressure or settlement under other drainage and loading conditions were almost identical.

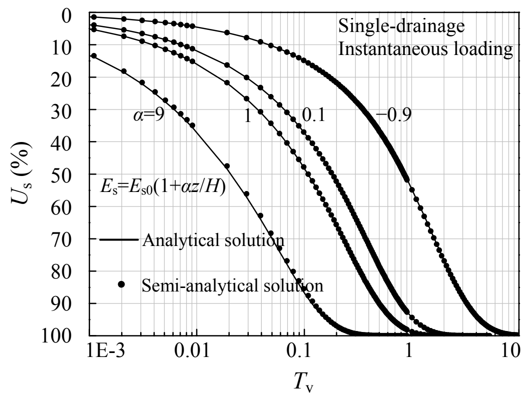

Fig.3 U_s vs T_v curves with different α in comparison with other solutions (single-layered)

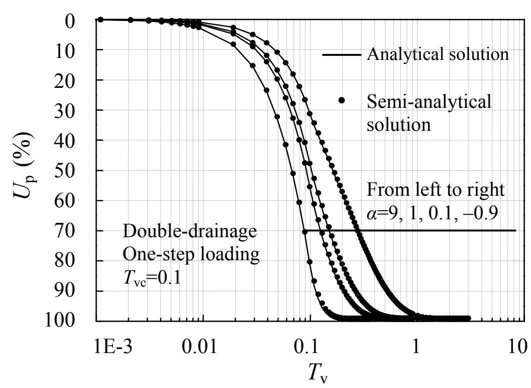

Fig.4 U_p vs T_v curves with different α in comparison with other solutions (single-layered)

Compared with the differential method proposed by Schiffman and Gibson (1964) and other numerical methods, the semi-analytical method can control the precision more easily and therefore speed up the calculation. The semi-analytical method can deal with every case in which k_v and m_v vary with depth, for example, k_v and m_v are polynomial function, exponential function or other nonlinear function of z. Furthermore, the solution to single-layered problem can be easily developed to solve the multi-layered soil problem.

CALCULATION ANALYSIS

The influence of the non-homogeneous characteristics of soil under instantaneous load on the consolidation process is analyzed and discussed below.

k_v and E_s are linear functions of z

1. Only E_s varies with z linearly

Fig.5 compares the solution obtained in a special case in which E_s varies with z linearly, e.g., $E_s=E_{s0}\times(1+\alpha z/H)$ with the corresponding solution of Terzaghi. In Fig.5 the average coefficient of consolidation $\bar{C}_v=\frac{1}{H}\int_0^H C_v \mathrm{d}z$ of four kinds of non-homogeneous soil is equal to one another and is equal to the consolidation coefficient of Terzaghi's solution. Fig.5 shows that when the modulus of compressibility varies greatly, the excess porewater pressure of the soil is greatly different from Terzaghi's solution for homogeneous soil calculated by the average value of the consolidation coefficient α=9 or α=−0.9 (the ratio of the compressibility modulus of top foundation to that of bottom foundation is 10 or 0.1). Furthermore, neglecting the non-homogeneous characteristic of the soil generally leads to over-evaluating the dissipation rate of the porewater pressure.

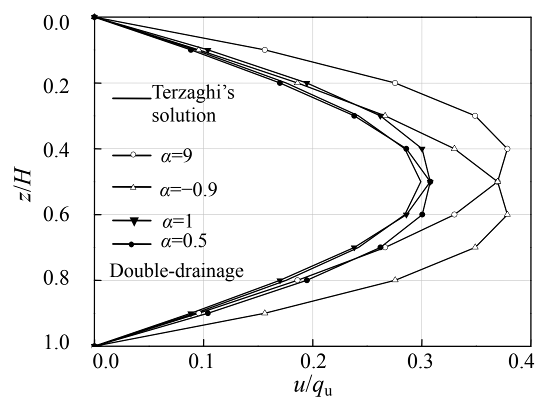

Fig.5 Isochrones of excess porewater pressure with linear E_s (t=100 d)

2. Only k_v varies with z linearly

Fig.6 compares the solution obtained in the special case in which k_v varies linearly (e.g., $k_v=k_{v0}\times(1+\beta z/H)$) with the corresponding solution of Terzaghi. When k_v varies greatly, the conclusion similar to that of Fig.5 can be derived.

In addition, in Fig.5 and Fig.6, the coefficients of consolidation corresponding to α=9, β=9 respectively both vary with depth linearly and the two variations are identical. The other three cases α=−0.9, 1, −0.5 corresponding to β=−0.9, 1, −0.5 are also shown in these two figures respectively. Comparison of the two figures shows that though the variation of c_v is identical, the distribution of the porewater pressure is different when k_v, m_v vary differently. Furthermore, when the variation extent of k_v and m_v is identical, the influence of k_v on the distribution of the porewater pressure is great. This conclusion can also be derived from the result of Terzaghi that the deviation of the porewater pressure of non-homogeneous foundation in Fig.6 is more significant than that in Fig.5.

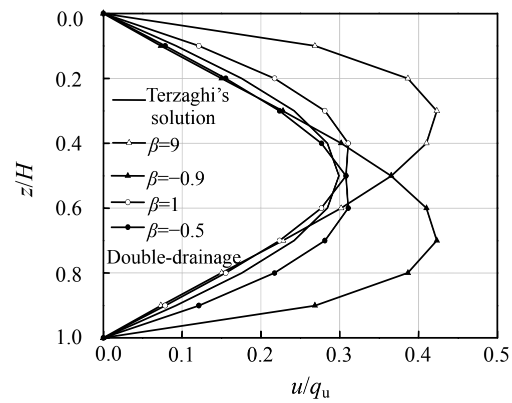

Fig.6 Isochrones of excess porewater pressure with linear k_v (t=100 d)

Both k_v and E_s are non-linear functions of z

In Fig.7, the consolidation curve is given based on the second case referred to in Section 3.1 and compared with the 1-D non-linear consolidation. To distinguish from the case that k_v and m_v (=1/E_s) vary non-linearly with both depth and time, the case when k_v and m_v vary non-linearly only with depth is still called consolidation of non-homogeneous soil. In this example, $N_\sigma=(\sigma_0'+q_u)/\sigma_0'=2.5$. Fig.7 shows that the smaller the ratio C_c/C_k, the closer are the curves of U_s. In two cases, different from non-linear consolidation, the rule of the variation of non-homogeneous soil consolidation with C_c/C_k is consistent, that is, from beginning to end, with the increase beginning to end, with the increase of C_c/C_k, the rate of consolidation also increases. For the degree of consolidation U_p defined in terms of the average porewater pressure, the same conclusion can be obtained.

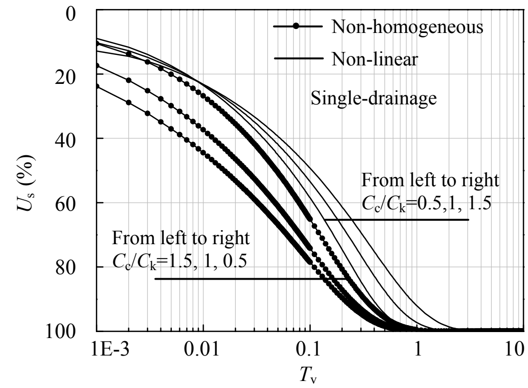

Fig.7 U_s vs T_v curves with different C_c/C_k of non-homogeneous soil in comparison with nonlinear consolidation soil (single-layered)

Based on non-linear and non-homogeneous assumptions, the problems of four-layer soil (Table 1) bellow can be studied respectively and the corresponding curves of the degree of consolidation are given in Fig.8.

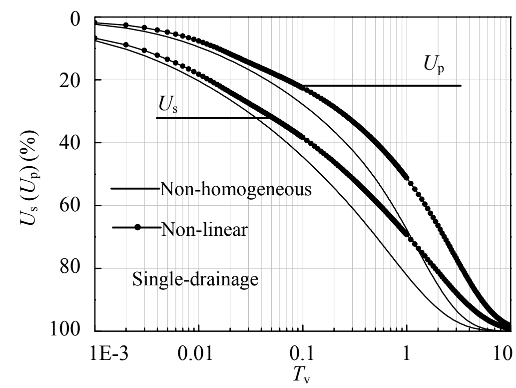

Fig.8 U_s (U_p) vs T_v curves of non-homogeneous soil in comparison with nonlinear consolidation soil (multi-layered)

Fig.7 and Fig.8 show that for either single-layered soil or multi-layered soil, the rate of consolidation calculated based on non-homogeneous assumptions is greater than that calculated based on non-linearity assumptions. Furthermore, Fig.8 shows that the average degree of consolidation U_s defined in terms of settlement is not equal to U_p defined in terms

Table 1 Geotechnical data on layered soil

i	C_{ci}	C_{ki}	k_{v1i} (10^{-9} m/s)	e_{1i}	σ_{1i}' (kPa)	γ_{sati} (kN/m^3)	h_i (m)
1	0.382	0.765	3.6280	1.393	100	18.31	3
2	0.521	0.525	0.8150	1.422	100	18.18	5
3	0.628	0.418	0.4342	1.501	100	17.91	8
4	0.286	0.358	1.1280	1.058	100	18.62	4

of average porewater pressure. This can also be derived from the definition of U_s and U_p. Generally, U_s is not equal to U_p for non-homogeneous foundation (except for the case when m_v does not vary with depth).

CONCLUSION

This work resolves by semi-analytical method the problem of 1-D consolidation of single-layered and multi-layered soil with coefficient of compressibility m_v and coefficient of permeability k_v varying with depth. Through comparison and calculation analysis, some conclusions can be obtained as follows:

(1) The 1-D consolidation problem of non-homogeneous soil is not dependent on the coefficient of volume compressibility m_v, although the coefficient of permeability k_v should be taken into consideration. When m_v, k_v vary greatly, there is great difference between the excess porewater pressure in non-homogeneous soil and the porewater pressure of Terzaghi's solution of homogeneous soil calculated by the average value of the consolidation coefficient.

(2) Except for the case when m_v does not vary with depth in single-layered soil, the degree of consolidation defined in terms of deformation is not equal to that defined in terms of the average porewater pressure for general non-homogeneous soil.

(3) For the 1-D consolidation problem of soft clay, the rate of consolidation calculated based on the assumptions that k_v and m_v distribute non-uniformly because the initial effective pressure variation with depth is greater than that calculated based on non-linearity assumptions. Furthermore, the smaller the value of C_c/C_k, the closer is the solution of non-homogeneous consolidation to the solution of non-linear consolidation.

References

Davis, E.H., Raymond, C.P., 1965. A non-linear theory of consolidation. *Geotechnique*, **15**(2):161-173.

Duncan, J.M., 1993. Limitations of conventional analysis of consolidation settlement. *Journal of Geotechnical Engineering ASCE*, **119**(9):1333-1359.

Jiang, W., Xie, K.H., Xia, J.Z., 2005. Analytical solution to 1-D consolidation of soft clay considering the modulus of compressibility varies with depth linearly. *Chinese Journal of Bulletin of Science and Technology*, (in Chinese, to be published).

Li, B.H., Xie, K.H., Ying, H.W., Zeng, G.X., 1999a. Semi-analytical solution of 1-D nonlinear consolidation considering the initial effective stress distribution. *Chinese Journal of Civil Engineering*, **32**(6):47-52 (in Chinese).

Li, B.H., Xie, K.H., Ying, H.W., Zeng, G.X., 1999b. Semi-analytical solution of one dimensional non-linear consolidation of soft clay under time-dependent loading. *Chinese Journal of Geotechnical Engineering*, **21**(3):288-293 (in Chinese).

Schiffman, R.L., Gibson, R.E., 1964. Consolidation of non-homogeneous clay layers. *Journal of the Soil Mechanics and Foundations Division, ASCE*, **4043**:1-30.

Xie, K.H., Pan, Q.Y., 1995. Theory of non dimensional consolidation of layered soils under variable loading. *Chinese Journal of Geotechnical Engineering*, **17**(5):80-85 (in Chinese).

Xie, K.H., Wen, J.B., Hu, H.Y., 2005. A study on one-dimensional consolidation of over-consolidated saturated soil. *Chinese Journal of bulletin of science and technology*, (in Chinese, to be published).

Xie, K.H., Xie, X.Y., Jiang, W., 2002. A study on one-dimensional nonlinear consolidation of double-layered soil. *Computers and Geotechnics*, **29**: 151-168.