Journal of Zhejiang University-SCIENCE C (Computers & Electronics)
ISSN 1869-1951 (Print); ISSN 1869-196X (Online)
www.zju.edu.cn/jzus; www.springerlink.com
E-mail: jzus@zju.edu.cn

Cooperative spectrum sensing in cognitive radio systems with limited sensing ability*

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Received Jan. 10, 2009; Revision accepted May 6, 2009; Crosschecked Nov. 30, 2009

Abstract: In cognitive radio systems, the design of spectrum sensing has to face the challenges of radio sensitivity and wide-band frequency agility. It is difficult for a single cognitive user to achieve timely and accurate wide-band spectrum sensing because of hardware limitations. However, cooperation among cognitive users may provide a way to do so. In this paper, we consider such a cooperative wide-band spectrum sensing problem with each of the cognitive users able to imperfectly sense only a small portion of spectrum at a time. The goal is to maximize the average throughput of the cognitive network, given the primary network's collision probability thresholds in each spectrum sub-band. The solution answers the essential questions: to what extent should each cognitive user cooperate with others and which part of the spectrum should the user choose to sense? An exhaustive search is used to find the optimal solution and a heuristic cooperative sensing algorithm is proposed to simplify the computational complexity. Inspired by this optimization problem, two practical cooperative sensing strategies are then presented for the centralized and distributed cognitive network respectively. Simulation results are given to demonstrate the promising performance of our proposed algorithm and strategies.

Key words: Cognitive radio, Cooperative spectrum sensing, Limited sensing ability, Throughput
doi:10.1631/jzus.C0910027 **Document code:** A **CLC number:** TN914.51

1 Introduction

Public mobile radio spectrum has become a scarce resource owing to the recent boom in wireless technologies. Cognitive radio (CR), a technique first proposed by Mitola and Maguire (1999) and Mitola (2000) and then promoted by the Federal Communications Commission (FCC, 2003), has attracted great attention recently as an important technology for dealing with an increasingly tense situation in spectrum use. The key feature of cognitive radio is that it allows unlicensed (cognitive) users to opportunistically exploit the licensed spectrum so as to fully utilize the available spectrum (Haykin, 2005). As cognitive radio has a lower priority than the licensed spectrum, it needs to be able to determine independently whether the spectrum is available at a particular time, and adjust its transmission and reception accordingly. Spectrum sensing has therefore become one of the major challenges confronting cognitive radio. The sensing performance of a single cognitive user is limited because of channel fading and shadowing effects. As a result, cooperative spectrum sensing, which can enhance sensing performance, has attracted considerable attention (Cabric *et al.*, 2004; Ghasemi and Sousa, 2005; Mishra *et al.*, 2006). In such a cooperative approach, every cognitive user performs sensing independently at first, and local sensing results are then exchanged to make a global

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* Project supported in part by the National Basic Research Program (973) of China (No. 2009CB320405), the National Natural Science Foundation of China (No. 60972057), and the National High-Tech Research and Development Program (863) of China (No. 2007AA 01Z257)

decision. It has been proved that by making use of the variability of signal strength at various locations, cooperative sensing can make the cognitive system robust in severe or poorly modeled fading environments without drastic requirements on individual cognitive radios (Cabric *et al.*, 2004).

A lot of research has been done on cooperative spectrum sensing in cognitive radio systems. Cooperative spectrum sensing is most effective when the cognitive users observe independent fading or shadowing (Cabric *et al.*, 2004; Liu and Shankar, 2006). Pawelczak *et al.* (2006) and Ghasemi and Sousa (2007) investigated performance degradation caused by correlated shadowing in terms of missing transmission opportunities. Beamforming and directional antennas have also been used to combat shadowing (Peh and Liang, 2007). Cabric *et al.* (2006) showed that cooperating with all users in the network does not necessarily achieve optimum performance. For optimum performance, the cooperative users with the highest primary user's signal-to-noise ratio (SNR) are chosen for collaboration, and constant detection rates and constant false alarm rates are used. Most previous studies on cooperative spectrum sensing were based on the assumption that each of the cognitive users has the full-band sensing ability for even a wide-band spectrum. However, owing to hardware and power constraints, the cognitive user's sensing ability is usually very limited. In fact, the cost to achieve wide-band spectrum sensing by a single cognitive user is quite high. It is realistic to assume that the cognitive user is able to sense only limited bandwidth of spectrum during a certain amount of time. This sensing limitation brings new challenges in the design of spectrum sensing/access strategies in cognitive radios.

To our knowledge, the study by Zhao *et al.* (2007) was the first to take into account the limited sensing ability of each user in a cognitive radio system. Based on the theory of partially observable Markov decision process (POMDP), they presented a cross-layer approach to incorporate the partial sensing results into the optimization of spectrum access. Jia *et al.* (2008) considered the possibility of sensing overhead because of the single-channel sensing ability in accessing a multi-channel opportunity, and formulated a sensing decision problem for a sequence of sensing processes as a well-defined optimal stopping problem. Most interestingly, Lai *et al.* (2008) developed a unified framework for the design and analysis of cognitive medium access protocol based on the classical bandit problem. They first assumed that the cognitive user has only single-channel sensing ability and then extended that to the multi-channel case. One common limitation of all the above studies is that cooperation among different cognitive users was not taken into account. Su and Zhang (2008) studied two different cooperative spectrum sensing policies: a random sensing policy and a negotiation-based sensing policy. However, as in the studies of Jia *et al.* (2008) and Lai *et al.* (2008), the detection error and the consequent performance degradation in the spectrum access stage were not considered, and therefore the proposed cooperative spectrum sensing strategies were still neither perfect nor practical.

More specifically, in a wide-band cooperative spectrum sensing scenario, with only partial sensing ability, each cognitive user has to decide to what extent it should cooperate with others and which part of the spectrum it should choose to sense. In fact, these two problems essentially mean the same thing. If more cooperation exists among the cognitive users (i.e., they tend to choose the same channels to sense), the sensing accuracy will be enhanced, which will lead to improved spectrum utilization in the spectrum access stage. However, the resultant spectrum access opportunities will also be reduced compared with the case of less cooperation in which they tend to sense different channels.

Motivated by the above observations, in this paper we will study the cooperative spectrum sensing problem to maximize the average throughput of a multi-user cognitive network sharing a wide-band spectrum with a primary network. The cognitive users are equipped with only limited sensing ability; i.e., they are able to conduct the spectrum sensing within only a small portion of spectrum each time and the partial sensing result is imperfect. Given the primary network's tolerable collision probabilities in each spectrum sub-band, the spectrum access performance metric (i.e., the average throughput of the cognitive network) is formulated as a function of the number of cooperative users. An exhaustive search is used to find the maximum average throughput, and a heuristic cooperative sensing algorithm is also proposed to simplify the computational complexity. Based on the solution of the optimal cooperative sensing problem, we present two cooperative sensing strategies for the centralized cognitive network and the distributed cognitive network, respectively.

2 System model and problem formulation

2.1 System description

We consider a wide-band spectrum consisting of M channels (Fig. 1). This spectrum is licensed to a primary network, and the primary network operates in a synchronous time-slotted fashion. The bandwidth of channel i is defined as B_i (i=1, 2, …, M). As the primary network does not use the whole spectrum all the time, we use θ_i ($0\le\theta_i\le1$, i=1, 2, …, M) to denote the probability that channel i is occupied by the primary network in one time slot. The value of θ_i depends on the channel allocation algorithm and the traffic statistics of the primary network, and does not change frequently.

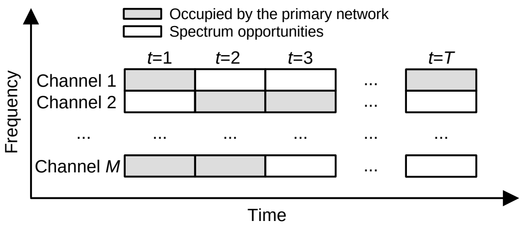

Fig. 1 Channel model

A cognitive network containing N users attempts to exploit the spectrum opportunities in these M channels. For the protection of the primary network, the cognitive network can transmit using only the idle channel and has to sense the spectrum before transmission. Let $S_i\in\{0 \text{ (idle)}, 1 \text{ (occupied)}\}$ be the availability of channel i in a time slot. Then the goal of spectrum sensing is to decide between the following two hypotheses:

$$S_i = \begin{cases} 0, & \mathrm{H}_0, \\ 1, & \mathrm{H}_1, \end{cases} \tag{1}$$

where hypothesis H_0 denotes the channel is idle, and H_1 represents the channel is occupied.

Owing to certain constraints, such as hardware limitations and time consumption limitations, it is difficult for a single sensor to achieve wide-band spectrum sensing. Without loss of generality, in the following discussion we assume that a cognitive user can sense only one channel at a time. The spectrum sensing performance is imperfect, and $P_{\text{false},i}$ and $P_{\text{miss},i}$ are used to denote the false-alarm probability and miss-detection probability, respectively, when a cognitive user chooses channel i to sense. Then the sensing performance of a single cognitive user for channel i can be specified by the receiver operating characteristics (ROC) curve, which gives $P_{\text{false},i}$ as a function of $P_{\text{miss},i}$:

$$P_{\text{false},i} = F_i(P_{\text{miss},i}). \tag{2}$$

In a practical system, the distance from the primary transmitter to the cognitive network is always much greater than the radius of the cognitive network. The average channel gain from the same primary user to each cognitive user is identical, and the average receiving SNRs are equal. Since the ROC function depends mainly on the detection approach and the receiving SNR, it is reasonable to assume that the ROC functions of the cognitive users sensing the same channel are identical.

Because of the imperfection of spectrum sensing, cognitive transmission may collide with a primary user and the primary transmission may suffer interference. We assume that the primary network is delay-insensitive, and no constraints are imposed on the average delay in the following discussion. Then, for the protection of the primary network, the collision probability of each channel should be constrained below a pre-determined threshold ζ_i (i=1, 2, …, M). The value of ζ_i depends on the channel condition and the throughput requirement of the primary network.

2.2 Cooperative spectrum sensing model

Since the sensing performance of a single cognitive user may be degraded because of channel fading and shadowing effects, cooperative spectrum sensing is used to increase the probability of detection. There are three successive stages in each time slot: the sensing stage, the reporting stage, and the transmission stage. At the sensing stage, each cognitive user performs spectrum sensing independently using the same detection approach. These local sensing results are then shared through a control channel at the reporting stage. For bandwidth limitation of the control channel (Cabric *et al.*, 2004), instead of using the raw data from sensing observations, only the final sensing results (H_0 or H_1) are transmitted. According to the local sensing results, the cognitive network can decide which channels are accessible. Finally, at the transmission stage, the cognitive network transmits using those idle channels according to the cooperative sensing result. Notice that this assumes that the time slot is sufficiently long to allow an appropriate detection time interval for the spectrum sensing.

To further limit interference to the primary user and to simplify the following discussion, the OR-rule (1-out-of-*n* rule) (Varshney, 1997) is used as the fusion rule. The following discussion can be easily extended to the cognitive system using the AND-rule as the fusion rule. The OR-rule can be described as follows: if any of the individual sensing results is H_1, the cooperative sensing result is H_1; otherwise, H_0 is decided. Let $\boldsymbol{n}=[n_1, n_2, ..., n_M]$ be the number of cognitive users choosing each channel to sense. All the n_i cognitive users sensing channel i use the same miss-detection probability $P_{\text{miss},i}$ and false-alarm probability $P_{\text{false},i}$. Although this assumption does not necessarily achieve optimal sensing performance, it leads to a simple and practical cooperative spectrum sensing algorithm. Furthermore, for simplicity of analysis, the correlation of the primary signal received by different cognitive users is not taken into account, and the n_i local sensing results are assumed to be independent of each other. The miss-detection and false-alarm probabilities of the cooperative sensing result for channel i can be obtained as follows:

$$Q_{\text{miss},i} = P_{\text{miss},i}^{n_i}, \tag{3}$$

$$Q_{\text{false},i} = 1-(1-P_{\text{false},i})^{n_i}. \tag{4}$$

2.3 Problem formulation

For a single cognitive user, when the collision probability threshold ζ_i is given, the optimal access policy is to trust completely the sensing results, and the optimal miss-detection probability is equal to ζ_i (Chen *et al.*, 2008). Similarly, this rule can be extended to the multi-user case, and the cognitive network can transmit using channel i if, and only if, its cooperative sensing result is H_0. Therefore, the miss-detection probability of the cooperative sensing result should be

$$Q_{\text{miss},i} = \zeta_i. \tag{5}$$

The expected miss-detection probability of the local sensing results can be obtained by

$$P_{\text{miss},i} = \zeta_i^{n_i^{-1}}. \tag{6}$$

The minimum false-alarm probability of the cooperative sensing result can then be given as

$$\min Q_{\text{false},i} = 1-\left(1-F_i(\zeta_i^{n_i^{-1}})\right)^{n_i}, \tag{7}$$

where $F_i(\cdot)$ is the corresponding ROC function for channel i.

Since the spectrum access strategy is not the focus of this paper, we assume that there is one cognitive user transmitting in one channel if it is decided as idle. In the centralized network, a control center can decide which cognitive user will transmit using the idle channel. In the distributed network, the cognitive users will follow a generalized version of the carrier sense multiple access/collision avoidance (CSMA-CA) protocol to access the idle channel. The cognitive transmission fails if transmission collision occurs. The cognitive transmission using channel i is successful only when this channel is idle and the cooperative sensing result is H_0. Therefore, the reward of the cognitive network sensing channel i can be defined as the average throughput, which is the product of the bandwidth B_i and the transmission successful probability:

$$r_i = B_i(1-\theta_i)(1-Q_{\text{false},i}) = B_i(1-\theta_i)\left(1-F_i(\zeta_i^{n_i^{-1}})\right)^{n_i}. \tag{8}$$

As described above, we focus on the following problem: given the probability θ_i that each channel is occupied, the collision probability threshold ζ_i, and the bandwidth B_i (i=1, 2, …, M), how does the cognitive network allocate the M channels to its N users in the cooperative spectrum sensing to optimize the subsequent spectrum access performance? Specifically, this allocation should be designed towards achieving the following two conflicting goals: (1) maximizing the average throughput of the cognitive network; (2) limiting the interference to the primary network caused by the cognitive transmission. Thus, the optimum cooperative sensing problem can be formulated as the following optimization problem:

$$\max \sum_{i=1}^{M} B_i(1-\theta_i)\left(1-F_i(\zeta_i^{n_i^{-1}})\right)^{n_i} \quad \text{s.t.} \quad \sum_{i=1}^{M} n_i \le N. \quad (9)$$

3 Analysis of the optimal cooperative sensing problem

In this section, we investigate the optimal cooperative sensing problem shown in Eq. (9). An exhaustive search is used to find an optimal solution, and a heuristic cooperative sensing algorithm is proposed to simplify the computational complexity.

3.1 The exhaustive search

In the optimization problem (Eq. (9)), the ROC function $F_i(\cdot)$ varies for different detection approaches and different signal statistics, and there is no general expression. Moreover, in most cases, even the closed-form expression of $F_i(\cdot)$ is hard to obtain. Therefore, it is difficult to find a general solution. As the optimization variable $\boldsymbol{n}=[n_1, n_2, ..., n_M]$ is a discrete vector, we can use an exhaustive search over each $\boldsymbol{n}$ to find the optimal solution. Considering the constraint on the total number of cognitive users, this exhaustive search has a computational complexity of $O\left(\sum_{l=1}^{N}\frac{(M+l-1)!}{(M-1)!l!}\right)$. In the following discussion, we will try to reduce this computational complexity in some special cases.

Let $G_i(x)$ denote the probability of correctly deciding channel i as idle:

$$G_i(x) = \left(1-F_i(\zeta_i^{x^{-1}})\right)^x. \quad (10)$$

As stated by the following proposition, when $F_i(\cdot)$ is convex, $G_i(x)$ has a useful property.

Proposition 1 $G_i(x)$ is nondecreasing if the ROC function $F_i(\cdot)$ is convex.

Proof See Appendix A.

The objective function in Eq. (9) is a nonnegative weighted sum of $G_i(x)$. According to Proposition 1, it is nondecreasing with an increasing number of cognitive users, N, if $F_i(\cdot)$ is convex. As we are maximizing a nondecreasing function, the optimal point is always at the constraint boundary, which means

$$\sum_{i=1}^{M} n_i \le N. \quad (11)$$

The optimization problem can be solved by an exhaustive search over all the possible combinations of $[n_1, n_2, ..., n_M]$ satisfying Eq. (11) and we have the following corollary:

Corollary 1 If the ROC function $F_i(\cdot)$ is convex, the optimal number of cooperative sensing users for each channel can be obtained through an exhaustive search with a computational complexity on the order of $O\left(\frac{(M+N-1)!}{(M-1)!N!}\right)$.

As the best ROC function is convex (van Trees, 2001), the computational complexity of the exhaustive search can be reduced according to Corollary 1 most of the time.

3.2 A heuristic algorithm

As discussed above, the optimal number of cooperative sensing users can be obtained through an exhaustive search. However, the computational complexity of an exhaustive search is too high when M and N are large. Thus, a simplified search algorithm is needed for practical implementation.

First, let us introduce Proposition 2.

Proposition 2 Let $\boldsymbol{n}(k)=[n_1(k), n_2(k), ..., n_M(k)]$ denote the solution of the optimization problem shown in Eq. (9) when the total number of the cognitive users $N=k$. If this optimization problem is convex, the optimal solution when $N=k+1$ can be expressed as

$$\boldsymbol{n}(k+1) = [n_1(k), ..., n_{i'}(k)+1, ..., n_M(k)],$$

where $i' = \arg\max_i \left(B_i(1-\theta_i)\left(G_i(n_i+1)-G_i(n_i)\right)\right)$.

Proof See Appendix B.

According to Proposition 2, we can obtain the following heuristic algorithm to simplify the search process:

Algorithm 1 The proposed heuristic cooperative spectrum sensing algorithm

n_i=0, $T(i)=B_i(1-\theta_i)[G_i(n_i+1)-G_i(n_i)]$ $\forall i$=1, 2, ..., M; j=0;
repeat
  $i' = \arg\max_i T(i)$; $n_{i'} = n_{i'}+1$; $j=j+1$;
  $T(i') = B_{i'}(1-\theta_{i'})\left(G_{i'}(n_{i'}+1)-G_i(n_{i'})\right)$;
until $j=N$

Based on Proposition 2, it is obvious that the solution of Algorithm 1 is optimal if the optimization problem shown in Eq. (9) is convex. Moreover, as presented above, the proposed algorithm calculates each $T(i)$ $\forall i$=1, 2, ..., M in the initialization stage. In any one of the following N repetitions, only $T(i')$ needs to be recalculated. As the computational complexity is caused mainly by the calculation of $T(i)$, the proposed algorithm has a computational complexity of the order of $O(M+N)$, which is much lower than the computational complexity of the exhaustive search.

4 Centralized and distributed cooperative spectrum sensing strategies

Based on the optimization problem analyzed above, in this section we will present two practical cooperative sensing strategies for the centralized and distributed cognitive network respectively.

4.1 Centralized cooperative spectrum sensing strategy

We first consider a centralized cognitive network (Fig. 2a). In such a system, a control center allocates one of the M channels to each cognitive user for cooperative spectrum sensing at the sensing stage. At the reporting stage, the local sensing results are transmitted to the control center, so that it can make a global decision for cooperative sensing. As the channel allocation decision is made by the control center, it can directly use the solution of the optimal cooperative sensing problem Eq. (9), and calculate the respective miss-detection probability $P_{\text{miss},i}$ for each cognitive user from Eq. (6).

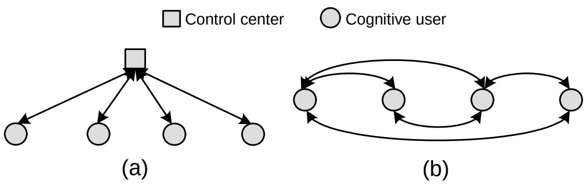

Fig. 2 Centralized and distributed system models
(a) A centralized cognitive network; (b) A distributed cognitive network

Let $\boldsymbol{n}$=[n_1, n_2, ..., n_M] be the solution of the optimal cooperative sensing problem. To maximize the average throughput of the cognitive network, n_i cognitive users should be set to sense channel i. Thus, the proposed centralized cooperative spectrum sensing strategy (CCSS) can be described as follows:

The optimal solution $\boldsymbol{n}$=[n_1, n_2, ..., n_M] for Eq. (9) is obtained by the control center at the beginning of each time slot.

1. The control center randomly chooses n_1 users from the total N users to sense channel 1, chooses n_2 users from the remaining $N-n_1$ users to sense channel 2, and so on. The expected miss-detection probability $P_{\text{miss},i}$ can be calculated using Eq. (10).

2. The sensing channel number i and the respective miss-detection probability $P_{\text{miss},i}$ are transmitted from the control center to each cognitive user using the control channel.

3. Local sensing is performed with the $P_{\text{miss},i}$ by each cognitive user.

4. The cognitive users transmit their local sensing results (H_0 or H_1) to the control center through the control channel.

5. The control center makes the global decisions for the M channels using the OR-rule.

4.2 Distributed cooperative spectrum sensing strategy

In a distributed cognitive network (Fig. 2b), there is no control center. The cognitive users have to choose the sensing channels and calculate the expected $P_{\text{miss},i}$ for local sensing by themselves. Moreover, local sensing results are exchanged among the N users, and the final decision is made by each cognitive user independently using the OR-rule. As all the cognitive users fuse the N local sensing results using the same fusion rule, the cooperative sensing results are identical.

In such a system, each cognitive user has to choose its sensing channel by itself, without any knowledge of other users' choices. Also, as the number of cognitive users choosing the same channel is unknown, the expected miss-detection probability cannot be directly calculated from Eq. (6). Obviously, CCSS given above is unsuitable for this system. Thus, a distributed cooperative sensing strategy is proposed in this subsection.

We assume that cognitive user j will choose channel i to sense with probability $p_{j,i}$ at each time slot, and the expected miss-detection probability for local sensing is $P_{\text{miss},ij}$. Then $\boldsymbol{p}_j$=[$p_{j,1}$, $p_{j,2}$, …, $p_{j,M}$] is the choosing rule of user j. For fairness and simplicity of implementation, a symmetric system is considered: all the cognitive users will follow the same choosing rule $\boldsymbol{p}=\boldsymbol{p}_1=\boldsymbol{p}_2=\ldots=\boldsymbol{p}_N$, and use the same miss-detection probability when sensing channel i: $P_{\text{miss},i} = P_{\text{miss},i1} = P_{\text{miss},i2} = \ldots = P_{\text{miss},iN}$.

Inspired by the solution of the optimization problem (Eq. (9)), we can set

$$p_i = n_i / N, \quad \text{for } i=1, 2, \ldots, M, \tag{12}$$

where $\boldsymbol{n}=[n_1, n_2, ..., n_M]$ is the optimal solution for Eq. (9). The number of cognitive users choosing channel i to sense is unknown to every cognitive user at the sensing stage. Therefore, given the sensing rule $\boldsymbol{p}$, how to calculate the expected miss-detection probability for local sensing becomes a question.

First, we have to compute the average miss-detection probability of the cooperative sensing result.

Proposition 3 Given the choosing rule $\boldsymbol{p}=[p_1, p_2, \ldots, p_M]$ and the miss-detection probability of the local sensing result $P_{\text{miss},i}$, the cooperative miss-detection probability for channel i can be expressed as

$$E(Q_{\text{miss},i}) = \frac{(P_{\text{miss},i} p_i + 1 - p_i)^N - (1-p_i)^N}{1-(1-p_i)^N}. \tag{13}$$

Proof Given a choosing rule $\boldsymbol{p}$, the probability that there are k_i cognitive users choosing channel i to sense can be expressed as

$$\binom{N}{k_i} p_i^{k_i} (1-p_i)^{N-k_i}. \tag{14}$$

The average miss-detection probability of the cooperative sensing result is

$$\begin{aligned} E(Q_{\text{miss},i}) &= \frac{\sum_{k_i=1}^{N} P_{\text{miss},i}^{k_i} \binom{N}{k_i} p_i^{k_i} (1-p_i)^{N-k_i}}{1-(1-p_i)^N} \\ &= \frac{(P_{\text{miss},i} p_i + 1 - p_i)^N - (1-p_i)^N}{1-(1-p_i)^N}. \end{aligned} \tag{15}$$

Thus, we have Proposition 3.

Then, for channel i, the maximum tolerable miss-detection probability is equal to the collision probability threshold ζ_i:

$$E(Q_{\text{miss},i}) = \zeta_i. \tag{16}$$

According to Proposition 3 and Eq. (16), the miss-detection probability of the local sensing result can be given as

$$P_{\text{miss},i} = \frac{1}{p_i}\left(\left(\zeta_i + (1-\zeta_i)(1-p_i)^N\right)^{\frac{1}{N}} + p_i - 1 \right). \tag{17}$$

Obviously, this distributed cooperative spectrum sensing strstegy (DCSS) performs worse than CCSS. However, as shown in Proposition 4, the performance gap of these two strategies can be ignored when the number of cognitive users is large enough.

Proposition 4 The proposed DCSS tends to have the same performance as the CCSS when the number of cognitive users $N\rightarrow\infty$.

Proof See Appendix C.

The details of DCSS are described as follows:

$\boldsymbol{p}=[p_1, p_2, \ldots, p_M]$ from Eq. (12) is obtained by each cognitive user at the beginning of the time slot.

1. Each cognitive user chooses its sensing channel according to $\boldsymbol{p}$, and calculates the expected miss-detection probability $P_{\text{miss},i}$ using Eq. (17).

2. Local sensing is performed with the miss-detection probability $P_{\text{miss},i}$ by each cognitive user.

3. Local sensing results (H_0 or H_1) are exchanged among the N cognitive users through the control channel.

4. Each cognitive user makes the final decisions for the M channels independently using the OR-rule.

5 Simulation results and analysis

In this section, simulation results are presented to evaluate the performance of the proposed cooperative spectrum sensing algorithms and strategies. Energy detection, a common method for detecting unknown signals in noise, was used in the following simulations. The radio propagation between the primary transmitter and any cognitive user was assumed to be affected by the independent stationary Rayleigh fading channel. The miss-detection probability and false-alarm probability of an energy detector in a

Rayleigh fading channel can be expressed as (Digham *et al.*, 2007)

$$P_{\text{detect}} = \exp\left(-\frac{\lambda}{2}\right)\sum_{i=0}^{T-2}\frac{1}{i!}\left(\frac{\lambda}{2}\right)^i + \left(\frac{1+\bar{\gamma}}{\bar{\gamma}}\right)^{T-1} \cdot \left[\exp\left(-\frac{\lambda}{2+2\bar{\gamma}}\right) - \exp\left(-\frac{\lambda}{2}\right)\sum_{i=0}^{T-2}\frac{1}{i!}\left(\frac{\lambda\bar{\gamma}}{2(1+\bar{\gamma})}\right)^i\right], \tag{18}$$

$$P_{\text{false}} = \exp\left(-\frac{\lambda}{2}\right)\sum_{i=0}^{T-1}\frac{1}{i!}\left(\frac{\lambda}{2}\right)^i, \tag{19}$$

where $\bar{\gamma}$ is the average receiving signal-to-interference-plus-noise ratio (SINR), T is the sampling number of the signal, and λ is the detection threshold. Without loss of generality, we assumed that T was a constant for every cognitive user, and all the simulations were performed with T=5. The ROC curves of energy detection with different γ are shown in Fig. 3.

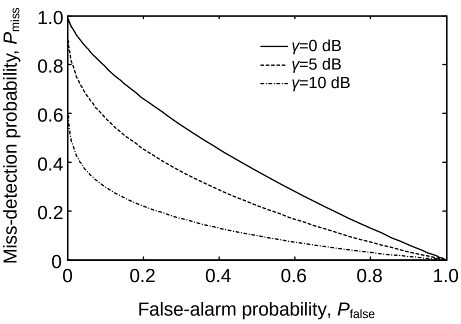

Fig. 3 The ROC curves of an energy detector in a Rayleigh fading channel

We first compared the performances of different cooperative spectrum sensing algorithms. Four different cooperative sensing algorithms were considered in the following simulations: (1) the optimal cooperative sensing algorithm by an exhaustive search for the optimization problem in Eq. (9), (2) the proposed heuristic algorithm, (3) the random-choosing cooperative sensing algorithm, which assumes that each cognitive user randomly chooses a channel to sense at each time slot, and (4) the minimum-choosing cooperative sensing algorithm, in which all the cognitive users will choose the channel with the minimum channel occupied probability. Since the ROC function of the energy detection is always convex, according to Corollary 1, the exhaustive search has a computational complexity of $O\left(\frac{(M+N-1)!}{(M-1)!N!}\right)$.

We considered four independent channels with the same bandwidth B=1 MHz and the same collision probability threshold ζ_i=0.1. Moreover, for simplicity, the receiving SINR γ was set to be equal for each user no matter which channel was sensed. Fig. 4 shows the average throughput of the cooperative sensing algorithms versus the receiving SINR γ. Each channel was occupied with probability: θ_1=0.3, θ_2=0.4, θ_3=0.5 and θ_4=0.6, and a cognitive system containing 8 cognitive users attempted to use the idle channels. It can be observed from Fig. 4 that the proposed heuristic algorithm had the same average throughput as the optimal algorithm, and outperformed the other two algorithms. Considering its lower computational complexity, the heuristic algorithm is more suitable for practical implementation. The minimum-choosing algorithm had the worst performance, and the performance gap becomes even larger as γ increased. This is because the minimum-choosing algorithm sets all the users to sense the same channel, so it approaches the performance threshold of the cooperative spectrum sensing (Ghasemi and Sousa, 2005) faster than the others.

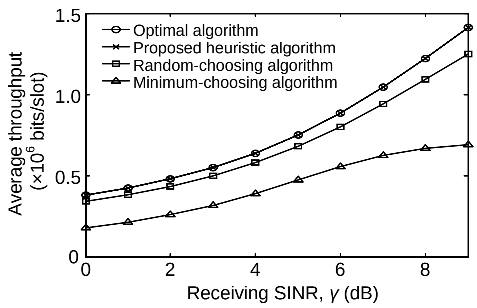

Fig. 4 The average throughput of the cognitive network versus the receiving signal to interference plus noise ratio (SINR)

Fig. 5 shows the impact of the channel occupied probabilities on the performance of each algorithm. A cognitive system containing 8 users was considered as in Fig. 4, and the receiving SINR γ=5 dB. We assumed that the sum of the probability that channel 1 or channel 2 was occupied was constant: θ_1+θ_2=0.8, and the probability that channel 3 was occupied was equal to the probability of channel 4: θ_3=θ_4=0.4. The average throughput is given while θ_1 increased from 0.4 to 0.8. Similar to the simulation results shown in Fig. 4, the heuristic algorithm had the same performance as the optimal algorithm, and the minimum-choosing algorithm performed worst. However, it should be noted that the heuristic and the optimal algorithms had larger average throughput when θ_1 increased, though the probability that any one of these channels was occupied was fixed. But the average throughput of the random-choosing algorithm did not change as θ_1 varied. This can be interpreted as follows: the heuristic and the optimal algorithms will adjust their allocation results according to the probabilities that each channel is occupied. Thus, more users are set to sense channels with lower probabilities, and the cognitive network can make full use of the sensing ability of each user. However, in the random-choosing algorithm, the mean number of the cognitive users choosing each channel to sense is equal and fixed. Recalling Eq. (9), the average throughput of the cognitive network using a random-choosing algorithm can then be treated as a function of the probability that any one of the four channels is occupied. Since this probability was set as a constant, the average throughput of the random-choosing algorithm also remained constant.

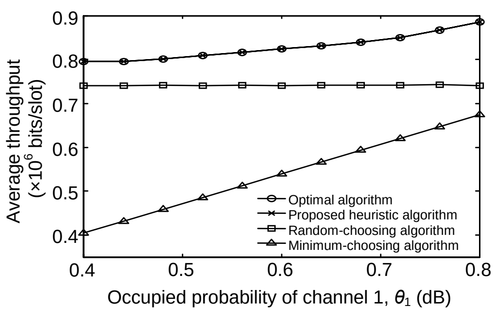

Fig. 5 The average throughput of the cognitive network versus the channel occupied probability

We then considered how the channels of interest were allocated to cognitive users by the optimal cooperative sensing algorithm with different system parameters. As the heuristic cooperative sensing algorithm had the same performance as the optimal cooperative sensing algorithm in the following simulations, its performance curves are omitted in Figs. 6 and 7. The allocation results with γ=3 dB and γ=10 dB are shown in Fig. 6. Four independent channels with the same bandwidth B=1 MHz were considered. The probabilities that each channel was occupied were set as: θ_1=0.2, θ_2=0.4, θ_3=0.6 and θ_4=0.8, and the number of cognitive users increased from 5 to 40. In both cases, the cooperative sensing algorithm always set more cognitive users to sense channels with lower θ. When N increased, the optimal number of cognitive users n_i also increased for all the channels, and the probability $p_i=n_i/N$ tended to be constant for all four channels. However, in the γ=10 dB case, the probability p_i tended to be nearly equal for each channel, while the differences among the four p_i's were still quite large in the γ=3 dB case. This is because the cooperative sensing gain for a single channel exhibits a 'law of diminishing returns' as the number of users is increased (Mishra *et al.*, 2006), and this has more effect on the cooperative sensing performance with a larger receiving SINR γ.

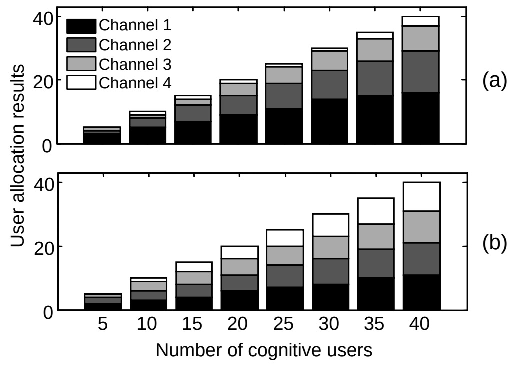

Fig. 6 The allocation results of the cooperative spectrum sensing algorithm versus the number of users
(a) γ=3 dB; (b) γ=10 dB

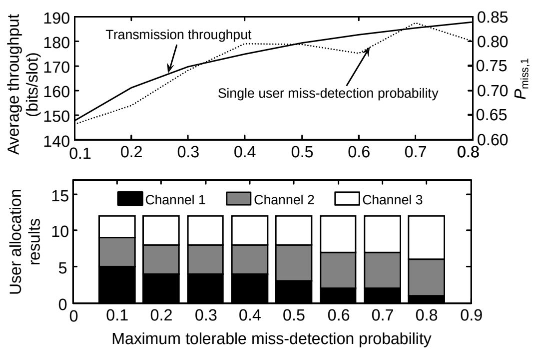

Fig. 7 The average throughput, the single user miss-detection probability of channel 1 and the allocation results versus the maximum tolerable miss-detection probability of channel 1

Fig. 7 shows the impact of the maximum tolerable miss-detection probabilities on the system performance. Three independent channels with the same bandwidth B=1 MHz were considered and each channel was occupied with the same probability θ=0.3. The cognitive network had 12 cognitive users and the receiving SINR γ=5 dB. The maximum tolerable miss-detection probability of channel 1 increased from 0.1 to 0.8 while the miss-detection probabilities of the other two channels always equaled 0.3. As ζ_1 increased, the cognitive network had larger average throughput, and tended to set fewer cognitive users to sense channel 1. However, the miss-detection probability of local sensing results for channel 1 did not monotonically increase with the increase of ζ_1. In fact, $P_{\text{miss},1}$ can be expressed as a function of ζ_1 and n_1 as shown in Eq. (6). Since n_1 decreases when ζ_1 increases, the curve of $P_{\text{miss},1}$ has a zigzag shape.

Finally, Fig. 8 shows the system performances of CCSS and DCSS which were proposed in Section 4. As a comparison, we also give the system performance of a random-choosing cooperative sensing strategy (RCSS) in a distributed cognitive network. In this DCSS, each user chooses one of the channels to sense with the same probability p=1/M, where M is the number of independent channels. The miss-detection probability of local sensing was set as Eq. (17). The simulations were performed with M=3, 10, 30, and the channel occupied probabilities were uniformly distributed on [0, 1]. The receiving SINR γ=0 dB. Clearly, CCSS had the best performance of all three strategies. The performance gap between CCSS and DCSS decreased when the number of cognitive users was sufficiently large. Moreover, even when the number of cognitive users was small, DCSS still outperformed RCSS.

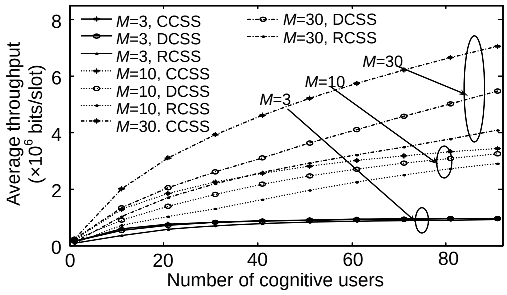

Fig. 8 The average throughput of different cooperative spectrum sensing strategies

CCSS: centralized cooperative spectrum sensing strategy; DCSS: distributed cooperative spectrum sensing strategy; RCSS: random-choosing cooperative sensing strategy

6 Conclusion

In this paper, we investigated the cooperative spectrum sensing performance in a cognitive network when the cognitive user has limited sensing ability. We formulated the optimal cooperative spectrum sensing problem with the constraint of transmission collision probability. An exhaustive search and a proposed heuristic algorithm were used to solve the optimization problem. Different cooperative sensing strategies for centralized and distributed cognitive networks were also studied. Simulation results were presented to show the promising performances of the proposed cooperative sensing algorithms and strategies. We hope this analysis will shed some light on the research and application of cooperative sensing in cognitive radio systems.

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Appendix A: Proof of Proposition 1

The first derivative of $G_i(x)$ is

$$\frac{\mathrm{d}G_i(x)}{\mathrm{d}x}=\left(1-F(\zeta_i^{\frac{1}{x}})_i\right)^{x-1}\cdot\left(\left(1-F(\zeta_i^{\frac{1}{x}})_i\right)\ln\left(1-F_i(\zeta_i^{\frac{1}{x}})\right)+\zeta_i^{\frac{1}{x}}F_i'(\zeta_i^{\frac{1}{x}})\ln\zeta_i^{\frac{1}{x}}\right). \tag{A1}$$

Clearly, the false-alarm probability $P_{\mathrm{miss},i}$ and miss-detection probability $P_{\mathrm{false},i}$ are less than or equal to 1, so we have

$$F_i(0)\le 1. \tag{A2}$$

Moreover, for a practical sensor, there is

$$F_i(1)=0. \tag{A3}$$

This is because the sensor can deny the presence of the signal all the time to reduce $P_{\mathrm{false},i}$ in the worst case.

As $F_i(\cdot)$ is convex, according to Eqs. (A2) and (A3), for any $\alpha\in[0, 1]$, we have

$$F_i(\alpha)\le\alpha F_i(1)+(1-\alpha)F_i(0)\le 1-\alpha, \tag{A4}$$

$$1-F_i(\alpha)=F_i(0)-F_i(\alpha)\ge-\alpha F_i'(\alpha). \tag{A5}$$

Thus, for any $x\ge 0$, we can obtain

$$\left(1-F_i(\zeta_i^{\frac{1}{x}})\right)\ln\left(1-F_i(\zeta_i^{\frac{1}{x}})\right)\ge-\zeta_i^{\frac{1}{x}}F_i'(\zeta_i^{\frac{1}{x}})\ln\zeta_i^{\frac{1}{x}}. \tag{A6}$$

Recalling that $P_{\mathrm{false},i}$ should be less than or equal to 1, we have

$$\left(1-F_i(\zeta_i^{\frac{1}{x}})\right)^{x-1}\ge 0. \tag{A7}$$

Taking Eqs. (A6) and (A7) into Eq. (A1), we find

$$\frac{\mathrm{d}G_i(x)}{\mathrm{d}x}\ge 0. \tag{A8}$$

Thus, we have Proposition 1.

Appendix B: Proof of Proposition 2

Let $\boldsymbol{l}=[l_1, l_2, ..., l_M]$, where $\sum_{i=1}^{M}l_i=k+1$. As $\sum_{i=1}^{M}l_i=\sum_{i=1}^{M}n_i(k+1)$, for any $\boldsymbol{l}\ne\boldsymbol{n}(k+1)$, we can find a j

satisfying $l_j>n_j$. According to the convexity of the optimization problem in Eq. (9), we have

$$G_j(l_j)-G_j(l_j-1)\le G_j(n_j)-G_j(n_j-1). \tag{B1}$$

Then, when the number of cooperative sensing users for each channel is $\boldsymbol{l}$, the average throughput can be expressed as

$$\begin{aligned}
&\sum_{i=1}^{M} B_i(1-\theta_i)G_i(l_i)\\
&=\sum_{i=1,\, i\ne j}^{M} B_i(1-\theta_i)G_i(l_i)+B_j(1-\theta_j)G_j(l_j-1)\\
&\quad +B_j(1-\theta_j)[G_j(l_j)-G_j(l_j-1)]\\
&\overset{\text{(a)}}{\le}\sum_{i=1,\, i\ne j}^{M} B_i(1-\theta_i)G_i(l_i)+B_j(1-\theta_j)G_j(l_j-1)\\
&\quad +B_j(1-\theta_j)[G_j(n_j+1)-G_j(n_j)]\\
&\overset{\text{(b)}}{\le}\sum_{i=1,\, i\ne j}^{M} B_i(1-\theta_i)G_i(l_i)+B_j(1-\theta_j)G_j(l_j-1)\\
&\quad +B_{i'}(1-\theta_{i'})[G_{i'}(n_{i'}+1)-G_{i'}(n_{i'})]\\
&\overset{\text{(c)}}{\le}\sum_{i=1}^{M} B_i(1-\theta_i)G_i(n_i)+B_{i'}(1-\theta_{i'})[G_{i'}(n_{i'}+1)-G_{i'}(n_{i'})]\\
&=\sum_{i=1,\, i\ne i'}^{M} B_i(1-\theta_i)G_i(n_i)+B_{i'}(1-\theta_{i'})G_{i'}(n_{i'}+1).
\end{aligned} \tag{B2}$$

The inequation (a) follows from Eq. (B1), (b) follows from the definition of i', and (c) follows from the assumption that $\boldsymbol{n}(k)$ is the optimal solution of Eq. (9) with $N=k$.

As $\sum_{i=1,\, i\ne i'}^{M} B_i(1-\theta_i)G_i(n_i)+B_{i'}(1-\theta_{i'})G_{i'}(n_{i'}+1)$ is the average throughput for $\boldsymbol{n}(k+1)$, we find that $\boldsymbol{n}(k+1)$ is the optimum solution of Eq. (9) for $N=k+1$ and Proposition 2 is proved.

Appendix C: Proof of Proposition 4

Let k_i be the number of cognitive users choosing channel i to sense in one time slot, $i=1, 2, ..., M$. All the users follow the same choosing rule $\boldsymbol{p}$ given by Eq. (12), and choose channel i with probability $p_i=n_i/N$. Therefore, according to Bernoulli's theorem (Vapnik, 2000), when the number of the cognitive users $N\to\infty$, k_i tends to be n_i.

However, when $N\to\infty$, the optimal number of cognitive users $n_i\to\infty$ and the probability p_i will become constant. Therefore, the limit of the expected miss-detection probability $P_{\text{miss},i}$ for CCSS is

$$\lim_{N\to\infty} P_{\text{miss},i}=\lim_{N\to\infty}\zeta_i^{\frac{1}{n_i}}=1, \tag{C1}$$

and the limit of $P_{\text{miss},i}$ for DCSS can be obtained as

$$\lim_{N\to\infty} P_{\text{miss},i}=\lim_{N\to\infty}\frac{\left(\zeta_i+(1-\zeta_i)(1-p_i)^N\right)^{\frac{1}{N}}+p_i-1}{p_i}=1. \tag{C2}$$

When $N\to\infty$, for channel i, k_i tends to be equal to n_i, and the expected miss-detection probabilities for the two strategies have the same limit. Therefore, the performance gap of these two strategies is decreasing to zero and Proposition 4 is proved.