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Joint bandwidth allocation and power control with interference constraints in multi-hop cognitive radio networks[*]

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Abstract: We investigate the bandwidth allocation and power control schemes in orthogonal frequency division multiplexing (OFDM) based multi-hop cognitive radio networks, and the color-sensitive graph coloring (CSGC) model is viewed as an efficient solution to the spectrum assignment problem. We extend the model by taking into account the power control strategy to avoid interference among secondary users and adapt dynamic topology. We formulate the optimization problem encompassing the channel allocation, power control with the interference constrained below a tolerable limit. The optimization objective with two different optimization strategies focuses on the routes rather than the links as in traditional approaches. A heuristic solution to this nondeterministic polynomial (NP)-hard problem is presented, which performs iterative channel allocation according to the lowest transmission power that guarantees the link connection and makes channel reuse as much as possible, and then the transmission power of each link is maximized to improve the channel capacity by gradually adding power level from the lowest transmission power until all co-channel links cannot satisfy the interference constraints. Numerical results show that our proposed strategies outperform the existing spectrum assignment algorithms in the performance of both the total network bandwidth and minimum route bandwidth of all routes, meanwhile, saving the transmission power.

Key words: Bandwidth allocation, Power control, Multi-hop cognitive radio networks, Spectrum management, Heterogeneous networks
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1 Introduction

Spectrum usage is concentrated on certain portions of the spectrum while the assigned spectrum ranging from 15% to 85% remains unutilized according to the Federal Communications Commission (FCC). The limited available spectrum and the inefficiency in spectrum usage necessitate a new communication paradigm to exploit the existing wireless spectrum opportunistically (Akyildiz *et al.*, 2004; 2006; Haykin, 2005). The cognitive radio techniques enable the usage of temporally free spectrum, referred to as spectrum holes or white space. NeXt generation (XG) communication networks, also known as dynamic spectrum access networks (DSAN) as well as cognitive radio networks, can provide high bandwidth to mobile users through dynamic spectrum access techniques in a heterogeneous wired/wireless network environment integrating cellular, wireless local area network (WLAN), worldwide interoperability for microwave access (WiMAX), mobile ad hoc networks (MANET), and public switched telephone network (PSTN) networks, etc.

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Ramanathan (1999) proposed the progress minimum neighbor first (PMNF) as a sequential heuristic solution to graph coloring for generalized channel assignment. Its objective is to minimize the total colors required to color all vertices. Zheng and Peng (2005) and Peng *et al.* (2006) proposed a mathematical model to reduce the spectrum allocation problem to color-sensitive graph coloring (CSGC) under a fixed topology, and described a set of centralized and distributed approximation algorithms to optimize the different utility functions. When the topology changes because of user movement the graph coloring algorithm needs to be repeated. Cao and Zheng (2005) used a local bargaining approach to adapt dynamic topology to approximate a new optimal assignment, taking prior allocation into account in a new spectrum assignment. In Peng *et al.* (2006) the secondary users can adjust only their transmission powers to avoid interference with primary users. However, these researches have not considered the power control problem to mitigate interference among secondary users and the interaction between power control and spectrum allocation (Cao and Zheng, 2005; Zheng and Peng, 2005; Peng *et al.*, 2006).

The secondary spectrum optimization problem was solved with the objective of maximizing the total transmitting rates of all secondary users while guaranteeing the quality of service (QoS) and the interference temperature below a threshold as in Xing *et al.* (2007), but the solution to the social spectrum optimization may be highly complex and is not feasible when all of secondary links are active. Joint link scheduling and power control was studied with the objective of maximizing network throughput in time division multiple access (TDMA) based multi-hop wireless networks, but it is not suitable for orthogonal frequency division multiplexing (OFDM) based cognitive radio networks (Tang *et al.*, 2006). Le and Hossain (2007) presented an admission control algorithm jointly with power control with an explicit interference protection for primary users and QoS guarantees for secondary users under a heavy load condition in a centralized cognitive code division multiple access (CDMA) wireless network. Obviously, the spectrum allocation approach is not suitable for the multi-hop case.

In this paper, we investigate the channel allocation model and power control technique with interference constraints in OFDM-based multi-hop cognitive radio networks, and most work on the OFDM-based channel allocation is based on fully-connected single hop wireless networks. To improve the frequency efficiency and optimize the system capacity, a joint bandwidth allocation and power control problem is formulated with the consideration of the interference constraints among secondary users by introducing a power control mechanism into the CSGC spectrum allocation model (Peng *et al.*, 2006), whose interference constraints are simply replaced by a distance relationship. Moreover, the spectrum allocation algorithms designed for general multi-hop networks topology (Peng *et al.*, 2006) are not efficient because of the link bottleneck or without the consideration that certain links support a multi-route. Finally, the heuristic solutions to our joint bandwidth assignment and power control problem are presented under different objective functions, which utilize channel reuse as much as possible and maximize the transmission power to improve the link bandwidth while satisfying the interference constraints. The contributions of this paper are three-fold:

1. We formulate the joint bandwidth allocation and power control problem by introducing the power control technique into a CSGC model and present the heuristic solutions to this NP-hard problem.

2. The optimization of channel allocation and power control focuses on the routes rather than the links as used in traditional approaches. It can avoid the link bottleneck and enhance the effectiveness of data transmission in multi-hop networks.

3. We obtain some important conclusions about power control and interference condition among co-channel links through deduction and simulations.

2 System architecture and problem model

2.1 System architecture

In a heterogeneous internetworking environment, the mobile users congested for scarce bandwidth, or transferred for the purpose of load balancing in case of unbalanced traffic distribution, or out of coverage of centralized access architectures, can form a secondary multi-hop cognitive radio network. They sense the unoccupied spectrum (Zhao *et al.*, 2007;

Chowdhury *et al*., 2008) individually or cooperatively and communicate with cognitive radio base stations through multi-hop relaying. The system architecture is shown in Fig. 1.

Suppose that all secondary mobile users communicate with a cognitive radio base station through a common control channel to exchange control information such as the bandwidth allocation and transmitting power level, etc. Moreover, the source node can always build an optimum multi-hop route to the cognitive radio base station (Xin, 2005). The cross-layer challenges about route selection and spectrum management will be discussed in our future work. The OFDM technique is adopted in the multi-hop cognitive radio networks because this technique has the advantage of feeding certain subcarriers with zero, thus exploiting the available but non-contiguous wireless spectrum. The frequency separation among subcarriers can be common or different (Xing *et al*., 2006).

2.2 Problem model and related works

We briefly introduce a spectrum assignment model (Peng *et al*., 2006) with some amendments to adapt our spectrum allocation policy, especially the variant of color-sensitive graph coloring.

Suppose that there are N secondary users to compete the use of M channels in multi-hop cognitive radio networks. They are indexed from zero to $N-1$, and $M-1$, respectively. Some definitions of the system parameters are given as follows:

1. Channel availability: $\boldsymbol{A}=\{a_{n,m}|a_{n,m}\in\{0, 1\}\}_{N\times M}$ is an $N\times M$ binary matrix representing the channel availability. $a_{n,m}=1$ if and only if channel m is available for user n.

2. Channel revenue: $\boldsymbol{B}=\{b_{n,m}\}_{N\times M}$ is an $N\times M$ matrix with $a_{n,m}$ representing the channel capacity that the nth user can achieve from the mth channel, which can be calculated as

$$b_{n,m}=W\log_2\left(1+\frac{gp}{\sigma^2+I}\right), \tag{1}$$

where W is the physical channel bandwidth, g is the transmission gain calculated through Eq. (2) (to appear in Section 3.1), p is the transmission power, σ^2 is the thermal noise power, and I is the total received interference power from the other links using channel m.

3. Interference constraint: $\boldsymbol{C}=\{C_{n,k,m}|C_{n,k,m}\in\{0, 1\}_{N\times N\times m}$ is an $N\times N\times M$ binary matrix, which represents the interference constraints among secondary users. $c_{n,k,m}=1$ when users n and k use channel m simultaneously and a conflict occurs. Conflict free channel assignment should satisfy all the interference constraints given by matrix $\boldsymbol{C}$.

By mapping each channel into a color and each node into a vertex, the spectrum allocation problem can be reduced to a CSGC model adopting the corresponding network topology (Peng *et al*., 2006). Fig. 2 illustrates an example of a CSGC algorithm. Secondary users I, II, III have a channel list available inside the brackets obtained by spectrum detection. These detected channels are identified by a natural number. The researcher obtained the interference constraints by the distance relationship among the transmitting nodes. For example, there is an edge marked by 1-color between secondary users I and III, which means that a collision will occur when they use channel 1 simultaneously. Similarly, there exists an

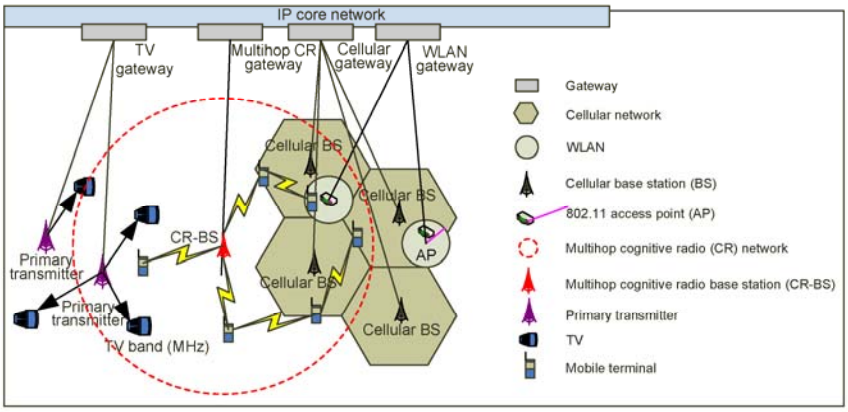

Fig. 1 Architecture of the multi-hop cognitive radio network in heterogeneous wired/wireless internetworking

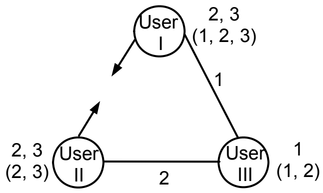

Fig. 2 An example of the color-sensitive graph coloring model for spectrum assignment

edge marked by 2-color between secondary users II and III. When these vertices are colored to allocate channels, the interference constraints must be satisfied. The natural numbers outside the brackets attached to each vertex represent the channels assigned to the corresponding node.

The CSGC model can be applied only to a fixed topology and need repeated computation under a dynamic topology, which will result in high algorithm complexity and low efficiency of spectrum assignment. For example, though there is no interference constraint between secondary users I and II under current topology, the channel allocation will bring collision immediately when they move towards each other (Fig. 2). Then, the QoS performance will degrade drastically. A power control technique can enhance the adaptation of spectrum allocation under dynamic topology. Moreover, when a power control strategy is adopted among secondary users, some interference constraints can be released. For example, the edge marked by 1-color can be cancelled when secondary users I and III control the transmission power. Therefore, the power control technique can improve the channel reuse in multi-hop cognitive radio networks, thus raising the frequency efficiency by channel reuse as much as possible. Our proposed spectrum assignment algorithm combined with a power control strategy will be illustrated specifically in the next section.

3 Joint bandwidth allocation and power control

3.1 Power control

The model of propagation gain used in our work is the same as in Shi and Hou (2007) and the transmission gain g_{kh} from nodes k to h can be formulated as

$$g_{kh}=d_{kh}^{-\rho}, \tag{2}$$

where d_{kh} is the physical distance between nodes k and h, and ρ is the path attenuation exponent.

In general, the transmission power p of the mobile terminal is bounded by zero and maximum power $P_{\max}$, i.e., $p\in[0, P_{\max}]$. Suppose that α, β are the received signal power threshold for successful transmission and the total received interference power threshold, respectively. Then, the maximum distance of a single hop is determined by the following formula:

$$d^{-\rho}\cdot P_{\max}\geq\alpha. \tag{3}$$

From Eq. (3) we know that the maximum distance between transceivers is

$$d\leq(P_{\max}/\alpha)^{1/\rho}. \tag{4}$$

Theorem 1 Suppose that two links $n\to f$ and $k\to h$ use the same channel, and the lengths of both links satisfy the maximum distance constraint given by Eq. (4), denoted as d_1 and d_2, respectively. Then, two links with accurate power control will not conflict with each other wherever the receivers f and h locate when the distance between the two transmitters n and k satisfies

$$d_{n,k}\geq\max\{d_1,d_2\}(\alpha/\beta)^{1/\rho}+\min\{d_1,d_2\}. \tag{5}$$

Proof For link $n\to f$, the transmission power with accurate power control can be calculated by the following formula:

$$d_1^{-\rho}\cdot p=\alpha. \tag{6}$$

Given the power p through Eq. (6), the interference range is determined by

$$(d_n^1)^{-\rho}\cdot p>\beta. \tag{7}$$

Combined with Eqs. (6) and (7), the interference range of transmission node n can be given by

$$d_n^1<(\alpha/\beta)^{1/\rho}d_1. \tag{8}$$

Similarly, we can calculate the interference range d_k^{I} of transmission node *k* for link *k*→*h*, which can be illustrated by Fig. 3.

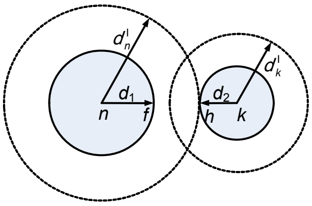

Fig. 3 Distance condition for two links without conflict
Two circles centered around each transmission node, with the inner circle (solid) representing the transmission range and the outer circle (dashed) representing the interference range

For simplicity, suppose $d_1 \geq d_2$. The same following result can be achieved for $d_1 \leq d_2$. If the distance between transmission nodes *n* and *k* satisfies Eq. (5), then the interference range of node *n* cannot contain node *h*, and the interference range of node *k* cannot also contain node *f* because

$$\left[(\alpha/\beta)^{1/\rho}-1\right]d_1 \geq \left[(\alpha/\beta)^{1/\rho}-1\right]d_2. \tag{9}$$

If Eq. (5) is not satisfied, then the interference range of node *n* will overlap with the transmission range of node *k* according to the triangle inequality theorem—it means that a conflict may occur.

Theorem 1 is valid when only two links exist. When the number of competing links is more than two, the received interference power should be accumulated. Applying Theorem 1 can greatly decrease the algorithm complexity of computing the interference constraints of the CSGC model. Otherwise, all the distances between the transceiver of all co-channel links should be computed, and the computation quantity will increase exponentially.

Corollary 1 Two links with the same receiving node cannot satisfy Eq. (5), and the transmission nodes of these two links cannot use the same channel.

Proof Suppose that nodes *n* and *k* have the same receiving node *h*. Then the maximum distance between these two transmitting nodes is achieved only when the three nodes are located in a straight line (Fig. 4). It can be represented as

$$d_{n,k}=d_1+d_2. \tag{10}$$

Because $(\alpha/\beta)^{1/\rho}>1,$ we have

$$\begin{aligned} d_{n,k}&=d_1+d_2=\max\{d_1,d_2\}+\min\{d_1,d_2\}\\ &\leq \max\{d_1,d_2\}(\alpha/\beta)^{1/\rho}+\min\{d_1,d_2\}. \end{aligned} \tag{11}$$

The interference range overlaps inevitably based on Theorem 1. This can be directly seen from Fig. 4. Therefore, nodes *n* and *k* cannot use the same channel to send data to node *h*.

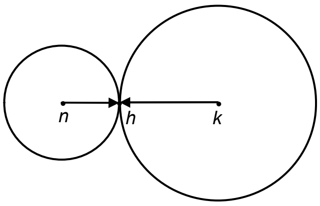

Fig. 4 Distance relationship for two links with the same receiving node

Corollary 2 Two links with the same transmitting node do not satisfy Eq. (5), and thus two links cannot use the same channel simultaneously.

The proof of Corollary 2 is easy and thus is not given here. Corollary 2 means that one user cannot use the same channel to send data to different receiving nodes. We will apply these results to the bandwidth allocation strategy for decreasing algorithm complexity and increasing channel reuse.

3.2 Bandwidth assignment policy with power control

Existing spectrum allocation algorithms have no power control strategy or consider only the interference constraints through the distance of neighboring nodes. Now we extend the CSGC spectrum assignment model by considering the co-channel link interference under accurate power control.

Suppose that each secondary user maintains an available channel list that is not used by primary users confirmed by spectrum cognition. Our allocation policy focuses on the route, but not the link, and the objective function is optimized by considering all the multi-hop routes to the cognitive radio base station. Suppose that there are *K* call requests, which necessitate *K* multi-hop routes to the cognitive radio base station, and that there are M_i links in the *i*th route. The link bandwidth is denoted as $R_{i,j}$, $1 \leq j \leq M_i$, which can be calculated as Eq. (1). Two optimization strategies are demonstrated specifically as follows:

1. Maximum total bandwidth strategy

The objective function of the maximum total bandwidth (MTB) strategy is to maximize the bandwidth summation of all K routes. Since the bandwidth of each multi-hop route is determined by the minimum bandwidth of all links in the route, the spectrum optimum assignment problem can be formulated as

$$\max \sum_{i=1}^{K} \min_{1\le j\le M_i} \{R_{i,j}\}, \tag{12}$$

$$\text{s.t. } \alpha d_{i,j}^{\rho} \le p_{i,j} \le P_{\max} \forall\, i=1,2,...,K;\ j=1,2,...,M_i, \tag{13}$$

$$I_{i,j} \le \beta, \tag{14}$$

where $d_{i,j}$, $p_{i,j}$ are the distance and the transmission power in the jth link of the ith route, respectively, and $P_{\max}$ is the maximum power threshold, $I_{i,j}$ is the total received interference power from other links using the same channels as this link. Therefore, the interference power constraints for co-channel links are given by Eq. (14). Suppose that the transceivers of this link are nodes k and h, respectively, and that channel m is used by this link. Then $I_{i,j}$ can be calculated as

$$I_{i,j} = \sum_{n\in \Lambda(m),\, n\neq k} g_{nh} p_n, \tag{15}$$

where $\Lambda(m)$ is the set of transmission nodes using channel m, and p_n is the transmission power of node n.

2. Minimum bandwidth optimization strategy

The objective function of the minimum bandwidth optimization (MBO) strategy is to maximize the bandwidth of one route with the minimum link bandwidth among K routes, which can be written as

$$\begin{aligned} &\max \min_{1\le i\le K} \min_{1\le j\le M_i} \{R_{i,j}\} \\ &\text{s.t. } \alpha d_{i,j}^{\rho} \le p_{i,j} \le P_{\max} \forall\, i=1,2,...,K;\ j=1,2,...,M_i, \\ &\quad I_{i,j} \le \beta. \end{aligned} \tag{16}$$

Obviously, the MBO strategy has the same constraints as the MTB strategy.

4 Heuristic approach

4.1 Spectrum assignment

The optimal channel assignment or its reduction to the graph coloring problem is known to be NP-hard. Therefore, the solutions of optimization problems Eqs. (12) and (16) can be solved only through heuristic based approaches to approximate the optimal solutions.

After adopting power control among secondary users to avoid interference, the interference constraint is related to the links attached to two transmitting nodes. Suppose that two nodes n and k have the same channel m in their channel lists, and l_n, l_k are the links attached to vertices n and k. When the distance between them satisfies Eq. (5), conflicts will not occur, i.e., $c(n, k, m, l_n, l_k)=0$; otherwise, $c(n, k, m, l_n, l_k)=1$. Based on the definition of conflict, we can define a new parameter denoted as $D_{n,m}$, i.e., m color-specific conflict degree about vertex n. It can be calculated as

$$D_{n,m} = \sum_{k=0,\, k\neq n}^{N-1} \sum_{l_n\in\phi(n)} \sum_{l_k\in\phi(k)} c(n,k,m,l_n,l_k)\, a_{n,m} a_{k,m}, \tag{17}$$

where $\phi(n)$ and $\phi(k)$ represent the link sets of transmitting nodes n and k, respectively. $a_{n,m}$ and $a_{k,m}$ are the channel availabilities for users n and k given by matrix A, respectively. Clearly, $D_{n,m}$ characterizes the conflict degree for all different links supported by vertex n in channel m. Different from Peng *et al.* (2006), in the definition of conflict degree $D_{n,m}$ in Eq. (17) the specific links supported by users n and k are taken into account; it results from the introduction of power control strategy.

To characterize the capability of data transmission for all mobile nodes before allocating channels to them, we define a parameter, denoted as CTD, which takes into account some primary influential factors including the number of channels available in its list, color-specific conflict degree, and the number of routes node supports. It can be formulated as

$$\mathrm{CTD}(n) = \frac{1}{r(n)} \sum_{m\in\zeta_n} \frac{1}{D_{n,m}+1}, \tag{18}$$

where $r(n)$ represents the number of routes in vertex n, and ζ_n is the set of available channels in the list attached to vertex n. Here, we do not consider color-specific interference; i.e., the interference constraint is the same for all channels available for one node. It is reasonable to assume that their interference levels are the same because the detected free channels for one node are often close to a certain radio frequency. These channels, not too widely separated, have the same features of fading and transmission. Then, the parameter CTD related to vertex n can be simplified as

$$\mathrm{CTD}(n)=\frac{|\zeta_n|}{r(n)(D_n+1)}, \tag{19}$$

where $|\zeta_n|$ is the number of channels available for vertex n.

Parameter CTD indicates the capability of data transmission in one node. With the definition CTD(n), we can label all K routes based on different strategies. This equals the definition of a priority of routes for coloring. The label rules according to the MTB or MBO strategy are formulated as follows:

$$\mathrm{label}_{\mathrm{MTB}}(i)=\max_{n\in\ell(i)}\mathrm{CTD}(n)\Big/\min_{n\in\ell(i)}\mathrm{CTD}(n), \tag{20}$$

$$\mathrm{label}_{\mathrm{MBO}}(i)=\min_{n\in\ell(i)}\mathrm{CTD}(n), \tag{21}$$

where $\ell(i)$ represents the set of M_i transmission nodes in the ith route. In our heuristic approach, channels will be allocated to all the routes according to the ascending order of label values of all routes. For the MTB strategy, Eq. (20) indicates the uniformity of data transmission capability of all nodes in the ith route. The route with the minimum label value, where all links have the most balanced capability of data transmission, will be assigned the first channel, and the other routes follow in turn. Eq. (21) defines the data transmission capability of the 'bottleneck' node in the ith route. Therefore, the route with the minimum bottleneck among the 'bottleneck' nodes of all routes will have the highest priority for coloring in the MBO strategy. Thus, the route with the minimum bandwidth is optimized by allocating the first channel to the worst route. For all links in the selected route of both MTB and MBO strategies, channels are allocated according to the ascending order of parameter CTD, and each time only one channel is allocated to each link. After each route is allocated one channel, the topology information, including the channel lists, interference constraints, and parameter CTD, etc., needs to be updated. The algorithm is then iteratively performed until all the channel lists attached to active nodes are empty.

4.2 Power calculation

Note that the channel assignment in our heuristic approach is performed under the transmission power $\alpha d_{i,j}^{\rho}$ in the jth link of the ith route, which is the smallest power that ensures the link is connected. In order to maximize the channel capacity, we need to maximize the transmission power, but bounded by the maximum power $P_{\max}$, while satisfying the interference constraints given by Eq. (14) for the co-channel links. For all the links using the same channel m, their transmission powers can be calculated through the following co-channel interference model.

In Fig. 5, N_m represents the number of links using channel m. As the transmission power of each link need to be maximized to raise link capacity, the transmission powers of those co-channel links can be formulated into the following multi-objective linear programming (LP) problem:

$$\max\ p_i \quad \forall\, i=1,2,\dots,N_m, \tag{22}$$

$$\text{s.t.}\ \begin{cases} p_2d_{2,1}^{-\rho}+p_3d_{3,1}^{-\rho}+\cdots+p_{N_m}d_{N_m,1}^{-\rho}\le\beta,\\ p_1d_{1,2}^{-\rho}+p_3d_{3,2}^{-\rho}+\cdots+p_{N_m}d_{N_m,2}^{-\rho}\le\beta,\\ \cdots,\\ p_1d_{1,N_m}^{-\rho}+p_2d_{2,N_m}^{-\rho}+\cdots+p_{N_m-1}d_{N_m-1,N_m}^{-\rho}\le\beta, \end{cases} \tag{23}$$

$$\text{and}\ \ \alpha d_{i,i}^{\rho}\le p_i\le P_{\max}\quad \forall\, i=1,2,\dots,N_m. \tag{24}$$

The multi-objective optimization problem above is NP-hard, but in most cases N_m is not more than 2 and the solution of Eq. (22) is easy to obtain. When N_m=1, its transmission power is obviously $P_{\max}$. When N_m≥3 we adopt the exhaustive search method to obtain the approximate optimum solutions.

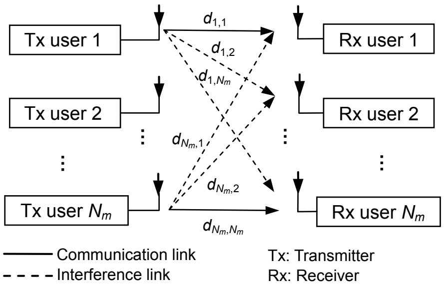

Fig. 5 Co-channel interference model for power calculation

For N_m=2, if we do not consider the upper bounded power $P_{\max}$, Eq. (23) can be represented as the matrix form

$$\begin{pmatrix} 0 & d_{2,1}^{-\rho} \\ d_{1,2}^{-\rho} & 0 \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \end{pmatrix}=\begin{pmatrix} \beta \\ \beta \end{pmatrix}. \tag{25}$$

It can be written as $\boldsymbol{DE}=\boldsymbol{\beta}$, where $\boldsymbol{D}$ represents the matrix $\begin{pmatrix} 0 & d_{2,1}^{-\rho} \\ d_{1,2}^{-\rho} & 0 \end{pmatrix}$, and $\boldsymbol{E}$ and $\boldsymbol{\beta}$ represent the vectors $\begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$ and $\begin{pmatrix} \beta \\ \beta \end{pmatrix}$, respectively. Then the power vector without upper bounded power can be calculated as

$$\boldsymbol{E}=\boldsymbol{D}^{-1}\boldsymbol{\beta}. \tag{26}$$

Finally, the transmission power considering the upper bounded power can be represented as

$$p_i=\min\{\boldsymbol{E}(i), P_{\max}\}\ \text{ for } i=1,2. \tag{27}$$

For $N_m \geq 3$, the way of obtaining solutions in the case of N_m=2 is not feasible. Similar to (Shi and Hou, 2007), the transmission power is discretized into a finite number of discrete levels between 0 and $P_{\max}$, and parameter Q represents the number of power levels. Therefore, the power interval of a single step, denoted by Δp, is $P_{\max}/Q$. Here, we gradually add $P_{\max}/Q$ transmission power to each link from the minimum transmission power $\alpha d_{i,i}^{\rho}$, and judge whether this link will result in the interference power of other links exceeding the interference threshold after it is added power. If this is affirmative, then we will stop adding power to this link. We add only one Δp transmission power to each link for each search. This process will stop until all co-channel links do not have additional power for controlling interference. The power search algorithm can be described specifically as follows:

```
**For** $i$=1 to $N_m$
  Calculate their minimum transmission power $p_i$ according to Eq. (6);
  label($i$)=0;   // zero means that this link can have added power level.
**End For**
**While** there are co-channel links whose label($i$)=0
  **For** $i$=1 to $N_m$ and label($i$)=0
    $p_i^{\text{new}} = p_i + \Delta p$;
    **If** $p_i^{\text{new}} > P_{\max}$ **then** $p_i^{\text{new}} = P_{\max}$;
    **End If**
    **For** $j$=1 to $N_m$ and $j \neq i$
      Calculate the total interference int_total to link $j$ after adding power;
      **If** int_total>$\beta$ then label($i$)=1; // label link $i$ to stop adding power to it.
        break;
      **End If**
    **End For**
    **If** label($i$)=0
      $p_i = p_i^{\text{new}}$; // the transmission power of the $i$th link is added.
      **If** $p_i^{\text{new}} = P_{\max}$ **then** label($i$)=1;
          // label link $i$ to stop adding power to it.
      **End If**
    **End If**
  **End For**
**End While**
```

In summary, we label all K routes according to our proposed MTB or MBO strategy, but not the links as in Peng *et al.* (2006), and then color each node of the selected route. The coloring process is performed iteratively until all routes have been allocated enough bandwidth or all of the color lists attached to mobile nodes are empty. The flow chart of spectrum assignment combined with power control is shown in Fig. 6.

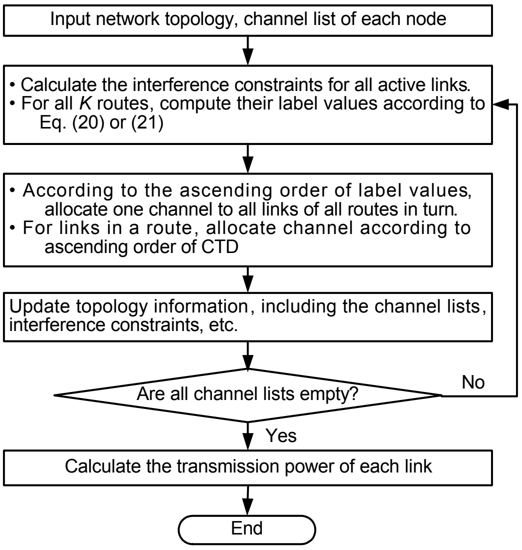

Fig. 6 Flowchart of spectrum assignment jointly with power control

5 Numerical results

We have conducted some numerical simulations to evaluate the performance difference between our proposed bandwidth allocation jointly with power control strategy and traditional algorithms. The referenced traditional algorithms are collaborative-max-sum-reward (CSUM) and collaborative-max-min-reward (CMIN) algorithms proposed by Peng *et al.* (2006). Obviously, they are link-oriented multi-hop spectrum assignment algorithms and do not take into account the factor of multi-route some nodes support. The power control technique among secondary users is not adopted in the referenced algorithms either, thereby the interference constraints in both of CSUM and CMIN approaches are obtained by comparing simply the distance relationship between secondary users.

5.1 Simulation configuration

We adopted a deterministic 20-node network topology defined in Shi and Hou (2007) to investigate the performance. MATLAB was used as the simulator to perform the recurrent search of bandwidth assignment and power calculation. The optimization was carried out by the central infrastructure such as the cognitive radio base station, and the ad hoc nodes implement only the multi-hop relay function.

We needed to expand the square area in 20-node network topology to 2000×2000 centered by the origin, so that all of the coordinates of 20 nodes were multiplied by 40 and then consistently subtracted by 1000. Source call nodes were 20, 3, 1, 18, 13, 17, 16, 19. They accessed the IP core network through the cognitive radio base station located in the origin. Some common simulation parameters are listed in Table 1.

Table 1 Simulation parameters

Parameter	Value
Subcarrier interval, B (kHz)	50
Noise power, σ^2	1.0×10^{-14}
Maximum transmission power, $P_{\max}$ (W)	0.1
Signal power threshold for successful transmission, α	5.0×10^{-13}
Interference power threshold, β	3.0×10^{-14}
Pathloss index, ρ	4
Power interval for a single step, Δp (W)	0.001
Maximum number of channels of each link	3

5.2 Simulation results

The network bandwidth performances varying with the number of channels and the maximum distance of a single hop are shown in Figs. 7 and 8, respectively.

We took the maximum transmission power as 0.1 W, which corresponds to the maximum communication distance of a single hop, about 668 m. Moreover, the total number of channels was changed in each simulation and the unoccupied channels detected by each node were produced at random. Fig. 7a shows that the total network bandwidth of all routes achieved from the proposed MTB strategy was larger than that from the proposed MBO strategy when the total number of channels changed. This is because the MTB strategy can maximize the network bandwidth gain by allocating channels to all of the links of one route symmetrically. Thus, link bandwidth is easier to contribute to the route bandwidth for symmetry. The MTB approach made the best use of channel resources. Moreover, both the MTB and MBO approaches outperformed the CSUM and CMIN algorithms because the route bandwidth is determined by the minimum bandwidth of all links in one route. However, the optimization of CSUM and CMIN approaches focusing on the links cannot efficiently achieve the bandwidth revenue of one route for the unbalance of link bandwidths, which makes the resource allocation ineffective. On the other hand, the power control technique can efficiently increase the channel reuse and achieve the route bandwidth gain. Fig. 7b validates that the efficiency of the MBO strategy is better than that of the MTB strategy for the optimization of minimum bandwidth. Clearly, the MTB optimization approach easily causes the starvation phenomenon. In contrast, the MBO optimization has the advantage of fairness among all routes because the bandwidth assignment in the MBO approach always favors the route in the weakest channel condition. As we expected, there are always routes obtaining no opportunity of data transmission according to both CSUM and CMIN algorithms. There are two reasons to account for this starvation phenomenon. Firstly, these two optimizations focusing on links cannot make an efficient contribution to the route bandwidth. Secondly, serious conflicts will occur when the power control technique is not adopted. Due to finite

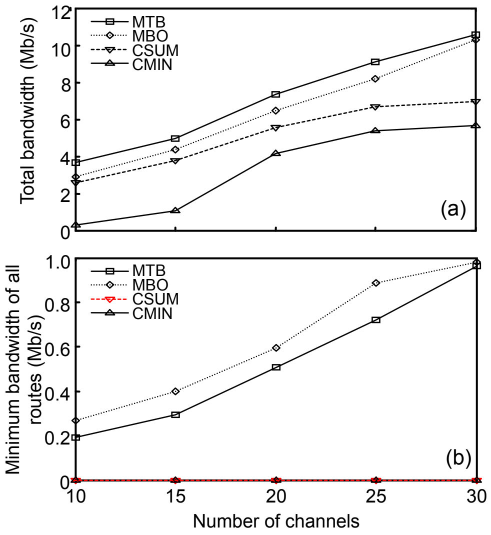

Fig. 7 Bandwidth revenue varying with the number of channels: (a) Total network bandwidth; (b) Minimum route bandwidth among all routes

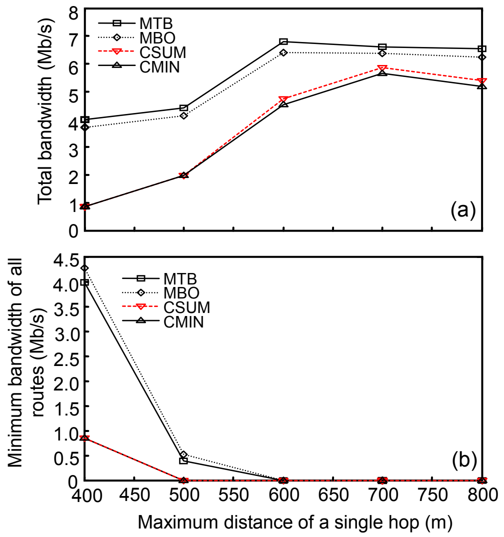

Fig. 8 Bandwidth revenue varying with the maximum distance of a single hop: (a) Total network bandwidth; (b) Minimum route bandwidth among all routes

channels and rigorous interference constraints, there exists a certain link of one route having no chance to have assigned channels.

Fig. 8a shows the total network bandwidth performance when the number of channels was fixed as 20 and the maximum communication distance of a single hop ranged from 400 to 800 m. It can be seen that the network bandwidth revenues were worsen when the distance of a single hop was less than 500 m. This is because some source call nodes cannot build multi-hop relay routes to the central cognitive radio base station due to the lack of relay nodes. The network capacity can be gradually improved by extending the communication distance of a single hop. However, when the distance of a single hop exceeded about 700 m, the network bandwidth gain cannot be achieved again for the extension of the communication scope. It will even result in performance degradation. This is because the larger the distance of a single hop is, the more serious the channel conflict will be. Actually, there is an optimum range of maximum communication distance of a single hop. It will be illustrated specifically in Theorem 2 in the Appendix. Moreover, our proposed MTB and MBO strategies can achieve a larger network capacity than both CSUM and CMIN approaches when the maximum distance of a single hop changes. The performance of the minimum route bandwidth is shown in Fig. 8b. This validates the better efficiency of our proposed bandwidth allocation approach combined with power control technique. Both MTB and MBO approaches outperformed the CSUM and CMIN algorithms from the minimum route bandwidth. When the distance of a single hop exceeded 600 m, there always existed at least one unconnected route in all the four approaches because a certain link of one route has no opportunity to have allocated channels. According to the simulation parameters and Theorem 2, the optimum range of the maximum single hop distance was evaluated as approximately [504, 624].

The distribution function of the transmission power was also investigated. The maximum transmission power was fixed as 0.1 W. The total number of channels was fixed as 20, and the free channels detected by secondary users were produced at random. The probability distribution of the transmission power of the secondary users is illustrated in Fig. 9. Clearly, the MBO approach saved the transmission power to some extent compared to the MTB optimization, because the MTB strategy should maximize the transmission power to increase the network capacity. However, the MBO strategy had the starting point of improving the link bottleneck. Therefore, it made the best use of the transmission power, leading to the fair bandwidth allocation among all routes. Moreover, the secondary users did not transmit data through the maximum transmission power 0.1 W in about 30 percent of links of all routes in both MTB and MBO approaches. Therefore, our proposed MTB and MBO optimization can save more transmission power and achieve more capacity gain in comparison with CSUM and CMIN algorithms.

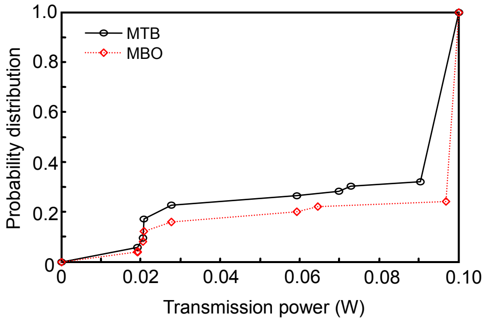

Fig. 9 Probability distribution function of the transmission power

6 Conclusion and future work

In this paper, we have analyzed the spectrum assignment algorithms based on the CSGC model in OFDM-based multi-hop cognitive radio networks. In order to improve the spectrum efficiency and adapt the dynamic topology to some extent, we propose a novel bandwidth optimization strategy combined with a power control technique under different optimization objective functions. We present the heuristic approaches to achieve approximately optimal solutions to this NP-hard problem. Another difference from the existing methods is that our optimization is aimed at the whole route rather than the links as in traditional channel allocation algorithms. The bandwidth allocation strategy under the minimum transmission power to guarantee the link connection takes into account the number of routes that one mobile node supports, the number of remaining channels, and the interference constraints of all co-channel links. The transmission power of each link is maximized to enhance the channel capacity while the interference is constrained below a certain threshold. Some important conclusions about bandwidth allocation and power control technique are achieved by deduction and simulation. Numerical results show that our proposed spectrum assignment strategies combined with a power control technique perform better than both CSUM and CMIN algorithms in performances of both total network bandwidth and minimum route bandwidth of all routes. Moreover, the proposed MTB and MBO approaches require less the transmission power compared to traditional algorithms.

Note that our proposed OFDM-based multi-hop cognitive radio networks architecture and spectrum assignment strategies are based on a centralized method. The future work will consider the bandwidth allocation and power control strategy in a distributed multi-hop network deployment and highly dynamic topology environment, and the mobility model (Ferrari and Tonguz, 2007) and spectrum handover mechanism will be applied to our bandwidth assignment policy to efficiently avoid conflicts.

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Appendix: Proof of Theorem 2

Theorem 2 The maximum communication distance of single hop should be within a certain range. Otherwise, the network performance will degrade drastically. For the square area with side length d, the range can be estimated as follows:

$$\left[2d/\sqrt{N\pi},\ d(\beta/\alpha)^{1/\rho}\sqrt{C/(\pi n\bar{h})}\right], \tag{A1}$$

where N is the number of all mobile users, and C is the number of available channels. n and $\bar{h}$ represent the number of calling nodes and the average number of hops for all routes, respectively. The range of the maximum single hop distance corresponds to the power range given the received signal power threshold α for a successful transmission.

Proof On one hand, the maximum communication distance of a single hop cannot be too short. Otherwise, the multi-hop route is hard to build. For the square area with side length d and N mobile users in this area, the covered average radius of each node can be evaluated as

$$d^2/N=\pi\bar{r}^2. \tag{A2}$$

Therefore, the average radius can be written as $\bar{r}=d/\sqrt{\pi N}$. In order to seek the relay node for the multi-hop route, the average minimum distance of a single hop must be larger than twice the average radius. It can be written as

$$d_{\min}\geq 2\bar{r}=2d/\sqrt{N\pi}. \tag{A3}$$

On the other hand, the maximum communication distance of a single hop cannot be too long. Otherwise, the interference range will be too large, resulting in an almost impossible channel reuse. The system performance will degrade drastically in multi-hop cognitive radio networks. From Eq. (8), we have the interference range as follows:

$$d_n^{\mathrm{I}}<(\alpha/\beta)^{1/\rho}d_{\max}, \tag{A4}$$

where $d_{\max}$ is the maximum distance of a single hop. Then, in order to have enough channels assigned to all links, we have the following formula with the consideration of channel reuse:

$$\frac{d^2}{\pi(d_n^{\mathrm{I}})^2}C\geq n\bar{h}. \tag{A5}$$

From Eqs. (A4) and (A5), we can evaluate the maximum communication distance of a single hop as

$$d_{\max}\leq d(\alpha/\beta)^{1/\rho}\sqrt{C/(\pi n\bar{h})}. \tag{A6}$$

Therefore, Eqs. (A3) and (A6) jointly bound the communication distance of a single hop.