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# New loop pairing criterion based on interaction and integrity considerations*

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**Abstract:** Loop pairing is one of the major concerns when designing decentralized control systems for multivariable processes. Most existing pairing tools, such as the relative gain array (RGA) method, have shortcomings both in measuring interaction and in integrity issues. To evaluate the overall interaction among loops, we propose a statistics-based criterion via enumerating all possible combinations of loop statuses. Furthermore, we quantify the traditional concept of integrity to represent the extent of integrity of a decentralized control system. Thus, we propose that a pairing decision should be made by taking both factors into consideration. Two examples are provided to illustrate the effectiveness of the proposed criterion.

**Key words:** Control structure design, Decentralized control, Interaction analysis, Variable pairing, Relative gain array
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## 1 Introduction

Control structure design (CSD) aims to make 'philosophical' decisions for a control system (Skogestad, 2004), such that plant operation can be maintained more easily at the optimal point. CSD includes two main aspects: the selection of variables (especially controlled variables (CVs)) and the selection of control configurations. Self-optimizing control (Skogestad, 2000) highlights the importance of the selection of CVs. By using feedback controllers, the appropriate selection of CVs results in near optimal operation in the face of uncertainties. Through local Taylor series expansion, Halvorsen *et al*. (2003) derived a simple method and an exact local method for the selection of CVs. Based on these strategies, various criteria and methods were proposed recently to select optimal CVs (Alstad *et al*., 2007; 2009; Kariwala, 2007; Kariwala *et al*., 2008; Hori and Skogestad, 2008).

Once manipulated variables (MVs) and CVs have been determined, there is an urgent need to select proper control configurations, and this is the main focus of this paper. The selection of control configurations refers to establishing the interconnecting relationships between MVs and CVs (Skogestad and Postlethwaite, 2005). A decentralized control structure decomposes the multi-input multi-output (MIMO) system into a number of smaller dimensional subsystems that can be controlled by much simpler controllers. Because of its marked advantages, such as simplicity and lucidity, decentralized control is highly favored and widely used in practice, and is still dominant in industrial applications. However, as the control loops are interacting through nonzero off-diagonal elements in the overall transfer function matrix, interaction is generally harmful to individual loop behavior. The principle of loop pairing is thus to choose those control loops that are minimally-linked. Interaction analysis is needed to instruct the decision making.

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Relative gain array (RGA) (Bristol, 1966) is the most popularly used loop pairing tool, and works effectively in substantial industrial applications (McAvoy, 1983; Shinskey, 1990). The most valuable advantage of RGA is that it provides a lot of interaction and stability information, while only steady state gain matrix is required and it is easy to calculate (Grosdidier *et al*., 1985). However, it suffers from several weaknesses that may sometimes lead RGA to make a wrong pairing choice.

During recent decades, RGA-based techniques have been developed in various ways to overcome these original shortcomings: (1) Process dynamics were considered. For instance, steady state gain matrix was replaced with the transfer function model (Witcher and McAvoy, 1977; Bristol, 1978; Tung and Edgar, 1981), the effects of steady state gain and response speed were balanced (Gagnepain and Seborg, 1982; Xiong *et al*., 2005), or controllers were employed in the process, so that interaction was analyzed dynamically (McAvoy *et al*., 2003; Schmidt and Jacobsen, 2003). (2) The RGA, combined with the Niederlinski index (NI) (Niederlinski, 1971) as a necessary stability condition, was used as an efficient tool for eliminating undesirable pairings. By exploring the intimate relationship between the sign of relative gain and stability problem, the concept of decentralized closed-loop integrity (DCLI) (Chiu and Arkun, 1990) was introduced to consider the integrity issue, namely whether the closed-loop system can remain stable as subsystem controllers are brought in and out of service (Skogestad and Postlethwaite, 2005). (3) Partial relative gain (PRG) (Häggblom, 1997) evaluates interaction when the system is under partial control and considers stability, but no clear criterion or systematic procedure was provided because of the complexity. The decomposed relative interaction analysis (DRIA) method (He *et al*., 2004) examines the interaction between an arbitrary loop and another. This approach is general enough until the system dimension reaches 4×4 or higher. Then, because DRIA still assumes the remaining subsystem loops are either all open or fully closed, omittance happens when more than one loop is left in the subsystem. (4) Since the meaning of distance between relative gain and desired unity is ambiguous, overall or normalized interaction measures (Zhu, 1996; Fatehi and Shariati, 2007) were derived to choose the most promising pairing when there are several different variable alternatives.

In this paper, we try to extract as much information as possible from the gain matrix from a steady state point of view. By enumerating all possible combinations of loop statuses, corresponding changed gains are obtained and their deviations are statistically analyzed. Relative expected gain (REG) is defined to represent the extent of deviation from expected gain. Variance index (VI) and expected integrity degree (EID) are both used to illustrate that a pairing decision is a multi-objective issue.

## 2 Preliminaries

Throughout this paper, we make the following assumptions: the system we are dealing with is square ($n$×$n$), open-loop stable, and has a nonsingular transfer matrix at steady state. Only a fully decentralized control structure will be considered; i.e., the small dimensional subsystems decomposed from MIMO systems are all single-input single-output (SISO).

Variable pairing is used to establish the one-to-one mapping relationship between manipulated and controlled variables, and then decentralized feedback control loops can be tuned and implemented. Traditional PI/PID controllers are adopted as feedback controllers. The diagonally paired structure is shown in Fig. 1.

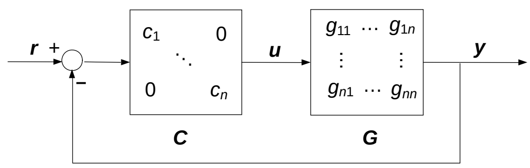


**Fig. 1 Diagonally paired decentralized feedback control structure**

The system is expressed by the following model with the Laplace operator $s$ omitted:

$$\boldsymbol{y} = \boldsymbol{G}\boldsymbol{u}, \tag{1}$$

where $\boldsymbol{y}$ and $\boldsymbol{u}$ are $n$-dimensional column vectors of outputs and inputs respectively, $\boldsymbol{G}$ is the transfer function matrix ($\boldsymbol{G}\in\mathbb{R}^{n\times n}$), and its individual element is denoted as $g_{ij}$. Since no dynamics are involved in this paper, the steady state gain matrix $\boldsymbol{G}(0)$ and element $g_{ij}(0)$ are expressed as $\boldsymbol{G}$ and $g_{ij}$ for simplicity respectively.

### 2.1 Relative gain array

The relative gain (Bristol, 1966) is defined as the ratio between the open loop gain $g_{ij}$ and the gain $\hat{g}_{ij}$ when all other loops are closed and in perfect control, i.e.,

$$\lambda_{ij} \triangleq \frac{\left.\dfrac{\partial y_i}{\partial u_j}\right|_{\text{all loops open}}}{\left.\dfrac{\partial y_i}{\partial u_j}\right|_{\text{all other loops in perfect control}}} = \frac{g_{ij}}{\hat{g}_{ij}}. \quad (2)$$

RGA is a matrix formed by $n^2$ relative gain elements and can be calculated (Grosdidier *et al*., 1985) as

$$\boldsymbol{\Lambda} = [\lambda_{ij}] = \boldsymbol{G} \otimes \boldsymbol{G}^{-\mathrm{T}}, \quad (3)$$

where the operator $\otimes$ is the element-by-element product and $\boldsymbol{G}^{-\mathrm{T}}$ is the inverse of $\boldsymbol{G}$ transposed.

The value of $\lambda_{ij}$ reflects the extent of loop $y_i$-$u_j$ affected by the actions of other loops. In particular, a value of unity implies the gain from $u_j$ to $y_i$ remains unchanged, whereas a negative value implies the gain experiences a sign change. The sign of the relative gain reflects important information for the stability problem, and pairing on a negative relative gain is typically not favored (Grosdidier *et al*., 1985).

### 2.2 Niederlinski index and decentralized closed-loop integrity

For a diagonally paired system, the Niederlinski index (NI) (Niederlinski, 1971) is defined as

$$\mathrm{NI} = \det \boldsymbol{G} \Big/ \prod\nolimits_{i=1}^{n} g_{ii}, \quad (4)$$

where the numerator is the determinant of the gain matrix and the denominator is the product of diagonal elements. For an arbitrarily paired system, the NI can be obtained by rearranging the order of inputs and outputs to be in diagonal form.

Assuming that the controllers contain integral action and that the loop gains are positive, a positive NI is required for a stable multiloop system. Conversely, a negative NI indicates that the closed system must be unstable. For 2×2 systems, a positive NI is a sufficient and necessary condition for the system to be stable.

Decentralized closed-loop integrity (DCLI) (Chiu and Arkun, 1990) was introduced to evaluate whether a control structure with integral controllers can remain stable in the face of one or more loop failures. We say a system has DCLI only if no instability occurs.

A number of necessary and/or sufficient conditions for DCLI have been given (Chiu and Arkun, 1990; Campo and Morari, 1994; He *et al*., 2005; Kariwala *et al*., 2005). In this paper, this binary concept of integrity will be quantified to assist in loop pairing.

### 2.3 RGA-plus-NI pairing rule and limitations

Based on the above, the mapping relationship of manipulated and controlled variables for a decentralized control system can be constructed. The most widely used RGA-plus-NI pairing rule is summarized as follows (Zhu, 1996):

1. RGA elements close to 1 are favored.
2. NI should be positive.
3. All paired RGA elements are positive.
4. Large RGA elements should be avoided.

This is an efficient tool to eliminate unviable pairing schemes and to pick out promising ones, while requiring only little effort in calculation. However, the simple calculation and incomplete consideration may lead to a wrong solution in some cases.

**Example 1** Consider the petlyuk distillation column given by Wolff and Skogestad (1995). The steady state gain matrix is

$$\boldsymbol{G} = \begin{pmatrix} 153.45 & -179.34 & 0.23 & 0.03 \\ -157.67 & 184.75 & -0.10 & 21.63 \\ 24.63 & -28.97 & -0.23 & -0.10 \\ -4.80 & 6.09 & 0.13 & -2.41 \end{pmatrix},$$

and the RGA is calculated as

$$\boldsymbol{\Lambda} = \begin{pmatrix} 24.5230 & -23.6378 & 0.1136 & 0.0012 \\ -48.9968 & 49.0778 & 0.0200 & 0.8990 \\ 38.5591 & -38.6327 & 1.0736 & 0.0000 \\ -13.0852 & 14.1827 & -0.2072 & 0.0998 \end{pmatrix}.$$

According to the RGA-plus-NI pairing rule, there are six viable pairing results whose relationships are expressed as $P_{1432}^{1} > P_{3412}^{2} > P_{1234}^{3} > P_{3214}^{4} > P_{1342}^{5} > P_{4312}^{6}$, where in $P^k$, the number $k$ means that the alternative is marked as the $k$th choice, and the number $j$ in the $i$th position of the subscript means output $y_i$ is paired with input $u_j$, and '>' reads 'better than' to prioritize viable pairings. The order is ranked by subjectively judging how the RGA elements are close to unity (Table 1).

**Table 1 Pairing results from applying the RGA-plus-NI rule to Example 1**

| Feasible pairing | RGA | NI |
|---|---|---|
| $P_{1432}^{1}$ | 24.5/0.9/1.1/14.2 | 0.0817 |
| $P_{3412}^{2}$ | 0.11/0.9/38.6/14.2 | 0.5089 |
| $P_{1234}^{3}$ | 24.5/49.1/1.1/0.1 | 0.0242 |
| $P_{3214}^{4}$ | 0.11/49.1/38.6/0.1 | 0.1506 |
| $P_{1342}^{5}$ | 24.5/0.02/5E−6/14.2 | 40.6360 |
| $P_{4312}^{6}$ | 1.2E−3/0.02/38.6/14.2 | 843.9023 |

To understand the limitations of the RGA-plus-NI rule, consider the following two situations:

1. Comparing pairing schemes $P^1$ and $P^3$, $y_1$ is controlled by $u_1$ and $y_3$ is controlled by $u_3$ for both $P^1$ and $P^3$. The differences are that in $P^1$, $y_2$ is controlled by $u_4$ and $y_4$ is controlled by $u_2$, while in $P^3$ this is reversed. Now assume that loop $y_1$-$u_1$ and loop $y_3$-$u_3$ are open and consider subsystem $\boldsymbol{G}^{11,33}$:

$$\boldsymbol{G}^{11,33} = \begin{pmatrix} 184.75 & 21.63 \\ 6.09 & -2.41 \end{pmatrix},$$

and RGA for $\boldsymbol{G}^{11,33}$ is calculated as

$$\boldsymbol{\Lambda}^{11,33} = \begin{pmatrix} 0.7717 & 0.2283 \\ 0.2283 & 0.7717 \end{pmatrix},$$

where $\boldsymbol{G}^{ij,kl}$ implies the submatrix of $\boldsymbol{G}$ with the $i$th, $k$th rows and $j$th, $l$th columns removed, and $\boldsymbol{\Lambda}^{ij,kl}$ is the corresponding RGA matrix of $\boldsymbol{G}^{ij,kl}$.

$\boldsymbol{\Lambda}^{11,33}$ implies the following facts: when loop $y_1$-$u_1$ and loop $y_3$-$u_3$ are open, for $P^1$, the gain of either loop in $\boldsymbol{G}^{11,33}$ will increase by a factor of 4.38 if the other loop closed perfectly; for $P^3$, the gain of either loop in $\boldsymbol{G}^{11,33}$ will increase by a factor of 1.30 if the other loop closed perfectly. Clearly, in such a situation the interaction in $P^3$ is less significant, and $P^3$ should be preferred to $P^1$. RGA overlooks the situations in which the system may be partially closed, leading to the conclusion that $P^1$ is prior to $P^3$. Note that, we are not claiming that $P^3$ is always better, because there are many other situations that may have other outcomes.

**Remark 1** When $y_1$-$u_1$ and $y_3$-$u_3$ are open, considering only subsystem $\boldsymbol{G}^{11,33}$ is reasonable because $\boldsymbol{y}$ and $\boldsymbol{u}$ in Eq. (1) are essentially the deviations from nominal values. The changed values in open loops have no effect on the gains considered.

2. For pairing scheme $P^1$, consider the subsystem $\boldsymbol{G}^{11}$:

$$\boldsymbol{G}^{11} = \begin{pmatrix} 184.75 & -0.1 & 21.63 \\ -28.97 & -0.23 & -0.1 \\ 6.09 & 0.13 & -2.41 \end{pmatrix},$$

and the RGA for $\boldsymbol{G}^{11}$ is calculated as

$$\boldsymbol{\Lambda}^{11} = \begin{pmatrix} 1.7270 & 0.1160 & -0.8431 \\ -1.2272 & 2.1867 & 0.0406 \\ 0.5002 & -1.3027 & 1.8025 \end{pmatrix}.$$

The negative element $(\lambda^{11})_{13}$ (−0.8431) means that for $P^1$, closing loops $y_3$-$u_3$ and $y_4$-$u_2$ will cause the gain from $u_4$ to $y_2$ to experience a sign change; i.e., instability occurs if $P^1$, which is recommended as the best pairing solution by the RGA-plus-NI rule, is adopted.

The two problems mentioned above are caused by incomplete considerations for a partially closed system. Häggblom (1997) tried to deal with this issue using partial relative gain. However, the complexity grows rapidly and analysis becomes difficult when the system dimensions become high. There is a lack of a systematic approach and a clearer criterion is needed.

## 3 Relative expected gain analysis

To investigate the interaction between loop $y_i$-$u_j$ and the remaining $(n-1)\times(n-1)$ subsystem $\boldsymbol{G}^{ij}$, consider subsystem $\boldsymbol{G}^{ij}$ when it is partially closed. Assume $m$ ($\forall m$=0, 1, ..., $n-1$) loops in $\boldsymbol{G}^{ij}$ closed and the other $n-m-1$ loops open. The interconnections between $\boldsymbol{y}$ and $\boldsymbol{u}$ are shown in Fig. 2, where $\boldsymbol{y}_{\rm CL}$, $\boldsymbol{u}_{\rm CL}$, $\boldsymbol{y}_{\rm OL}$, and $\boldsymbol{u}_{OL}$ ($\boldsymbol{y}_{CL}$, $\boldsymbol{u}_{CL}\in\mathbb{R}^m$; $\boldsymbol{y}_{OL}$, $\boldsymbol{u}_{OL}\in\mathbb{R}^{n-m-1}$) are vectors of controlled and manipulated variables in closed and open loops in $\boldsymbol{G}^{ij}$, $\boldsymbol{g}_{(\cdot)(*)}$ is the submatrix of $\boldsymbol{G}$, whose rows and columns are specified by $\boldsymbol{y}_{(\cdot)}$ and $\boldsymbol{u}_{(*)}$ respectively.

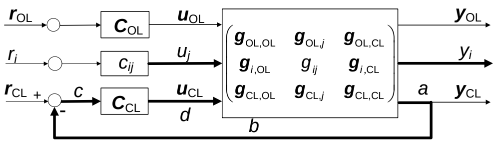


**Fig. 2 Decomposition of a partially closed system for loop $y_i$-$u_j$**

### 3.1 Partial gain

The impact from $u_j$ to $y_i$ transmits in two directions along the bold lines in Fig. 2: '$u_j$-$g_{ij}$-$y_i$' and '$u_j$-$\boldsymbol{g}_{CL,j}$-$a$-$b$-$c$-$\boldsymbol{C}_{CL}$-$d$-$\boldsymbol{g}_{i,CL}$-$y_i$'. This can be described in subsystem

$$\boldsymbol{G}^{OL}: \begin{pmatrix} y_i \\ \boldsymbol{y}_{CL} \end{pmatrix} = \begin{pmatrix} g_{ij} & \boldsymbol{g}_{i,CL} \\ \boldsymbol{g}_{CL,j} & \boldsymbol{g}_{CL,CL} \end{pmatrix} \begin{pmatrix} u_j \\ \boldsymbol{u}_{CL} \end{pmatrix}, \qquad (5)$$

where $\boldsymbol{G}^{(\cdot)}$ is the shorthand notation of $\boldsymbol{G}^{(\cdot),(\cdot)}$, and the superscript denotes 'removed' whereas the subscript denotes 'included'.

Assume $\boldsymbol{y}_{CL}$ are in perfect control at steady state. The gain $\dot{g}_{ij}$ from $u_j$ to $y_i$ is equivalent to $\hat{g}_{ij}[\boldsymbol{G}^{OL}]$ and can be calculated as

$$\begin{aligned} \dot{g}_{ij} &= \hat{g}_{ij}[\boldsymbol{G}^{OL}] = g_{ij} \big/ \lambda_{ij}[\boldsymbol{G}^{OL}] \\ &= \frac{g_{ij}}{(g_{ij} \det \boldsymbol{G}_{CL} / \det \boldsymbol{G}_{OL})} = \frac{\det \boldsymbol{G}^{OL}}{\det \boldsymbol{G}_{CL}}, \end{aligned} \qquad (6)$$

where $\hat{g}_{ij}[\boldsymbol{G}^{OL}]$ is the gain from $u_j$ to $y_i$ when all other loops in $\boldsymbol{G}^{OL}$ closed perfectly, and $\lambda_{ij}[\boldsymbol{G}^{OL}]$ is the corresponding RGA element of $\boldsymbol{G}^{OL}$.

Since $\boldsymbol{G}^{OL}$ is composed of loop $y_i$-$u_j$ and $\boldsymbol{G}_{CL}$, Eq. (6) indicates that for loop $y_i$-$u_j$, $\dot{g}_{ij}$ is dependent on $\boldsymbol{G}_{CL}$; i.e., $\dot{g}_{ij}$ depends on how many and which CVs and MVs are in the closed subsystem. Note that $\dot{g}_{ij}$ is independent of the internal pairing relationships in $\boldsymbol{G}_{CL}$. This explains why the interaction measure $\lambda_{ij}$ in RGA analysis is independent of how the other $n-1$ loops are paired, because only two extremes ($\boldsymbol{G}_{CL}=\varnothing$ and $\boldsymbol{G}_{CL}=\boldsymbol{G}^{ij}$) are considered.

Hence, in contrast, we find interaction is pairing dependent. One can easily consider that when the closed loop number $m$=1, $\boldsymbol{y}_{CL}$ and $\boldsymbol{u}_{CL}$ reduce to be scalars $y_{CL}$ and $u_{CL}$ respectively. Thus, $y_{CL}$ is paired with $u_{CL}$, and obviously $\dot{g}_{ij}$ varies with different pairs $y_{CL}$-$u_{CL}$. Pairing dependency is an important conclusion and the following discussions are based on the precondition that a pairing scheme $P^k$ has been assumed.

For a paired loop $y_i$-$u_j$ in $P^k$, variables in $\boldsymbol{y}^i$ and $\boldsymbol{u}^j$ can now be selected into $\boldsymbol{G}_{CL}$ couple by couple. Therefore, the number of different scenarios can be counted by summation of all possible combinations for $m$ =0, 1, ..., $n-1$:

$$\sum_{m=0}^{n-1} C_{n-1}^{m} = 2^{n-1}, \qquad (7)$$

where $C_{n-1}^{m}$ is the combination of $m$ out of $n-1$, which equals $(n-1)!/[m!(n-m-1)!]$. The right side of Eq. (7) can also be interpreted as the other $n-1$ loops' individual binary status. Therefore, there are $2^{n-1}$ different $\dot{g}_{ij}$'s that should be considered. The following gives a clear definition for this concept:

**Definition 1** Given a pairing scheme $P^k$ for multivariable process $\boldsymbol{G}$, partial gain $\dot{g}_{ij}(\boldsymbol{y}_{CL})$ is defined as the gain from $u_j$ to $y_i$ when the system is partially closed and achieves perfect control, where $y_i$-$u_j$ is one of the paired loops and $\boldsymbol{y}_{CL}$ ($\boldsymbol{y}_{CL}\in\mathbb{R}^m$) describes the set of controlled variables in closed loops.

The value of $\dot{g}_{ij}(\boldsymbol{y}_{CL})$ can be calculated using Eq. (6) by enumerating different $\boldsymbol{y}_{CL}$'s. In particular, $\dot{g}_{ij}(\varnothing)$ equals $g_{ij}$ and $\dot{g}_{ij}(\boldsymbol{y}^i)$ equals $\hat{g}_{ij}$. Let $\boldsymbol{A}_i$ be the set of all partial gains for an arbitrary paired loop $y_i$-$u_j$. As discussed above, $\boldsymbol{A}_i$ contains $2^{n-1}$ elements. Each loop has a partial gain set, and thus we have a total of $n$ partial gain sets $\boldsymbol{A}_i$ ($i$=1, 2, ..., $n$).

### 3.2 Expected gain and relative expected gain

We want to measure the interaction between loop $y_i$-$u_j$ and the other $n-1$ loops. Recall that in RGA analysis, the ratio between $g_{ij}$ and $\hat{g}_{ij}$ is used. The closer the ratio approaches unity, the less interaction is involved. This is quite efficient if there are only two gains. However, because we now have $2^{n-1}$ gains in $\boldsymbol{A}_i$, emphasizing one specific gain in $\boldsymbol{A}_i$ as the base case is no longer sound. A new index is needed to evaluate the extent to which they approach each other in closeness.

In total, there are $n$ loops and $2^n$ different scenarios (we use 'scenario' to mean any possible combinations of loop statuses). In practice, these scenarios may appear with different frequencies and should be treated unequally depending on their individual weighting factor. Determining these weighting factors is a very exhaustive task and lacks guidance in the design phase. To simplify the problem, we assume that any loop switches its own status (i.e., open or closed) without affecting the statuses of others, and then the total scenario profile obeys an independent distribution. Therefore, only a vector containing $n$ independent variables is needed to describe $2^n$ scenarios, named the 'loop-open probability vector',

$$\boldsymbol{M} \triangleq [\mu_i]^{\mathrm{T}} = [\mu_1 \cdots \mu_n]^{\mathrm{T}}, \tag{8}$$

and correspondingly, the 'loop-closed probability vector'

$$\bar{\boldsymbol{M}} \triangleq [\bar{\mu}_i]^{\mathrm{T}} = [\bar{\mu}_1 \cdots \bar{\mu}_n]^{\mathrm{T}}, \tag{9}$$

s.t.

$$\mu_i + \bar{\mu}_i = 1,\ \mu_i \ge 0,\ \bar{\mu}_i \ge 0,\ i = 1, 2, \ldots, n, \tag{10}$$

where $\mu_i$ and $\bar{\mu}_i$ are the open and closed probability, respectively, of the $i$th loop (loop $y_i$-$u_j$). Then the probability of $\dot{g}_{ij}(\boldsymbol{y}_{\mathrm{CL}})$ can be calculated as

$$\mu_{ij}(\boldsymbol{y}_{\mathrm{CL}}) = \prod_{\mu_k \in \boldsymbol{M}_{\mathrm{OL}}} \mu_k \cdot \prod_{\bar{\mu}_l \in \boldsymbol{M}_{\mathrm{CL}}} \bar{\mu}_l, \tag{11}$$

where $\boldsymbol{M}_{\mathrm{OL}}$ and $\boldsymbol{M}_{\mathrm{CL}}$ are specified by $\boldsymbol{y}_{\mathrm{OL}}$ and $\boldsymbol{y}_{\mathrm{CL}}$.

**Remark 2** The physical meaning of $\mu_i$ can be interpreted as how often the $i$th loop functions in manual mode are switched by a field operator, and also the probability of sensor/actuator failure, assuming that the corresponding controller is switched to manual mode when a loop failure occurs (Chiu and Arkun, 1990).

By introducing probability factors, the definitions of expected gain and relative expected gain can be derived as follows.

**Definition 2** For a paired loop $y_i$-$u_j$, the expected gain is defined as the nominal value of gain when the loop works at the expected virtual point. Mathematically, expected gain is equivalent to the probability-weighted sum of all partial gains

$$g_{ij}^{o} = E(\dot{g}_{ij}) = \sum_{\boldsymbol{y}_{\mathrm{CL}} \subseteq \boldsymbol{y}^i} \left( \mu_{ij}(\boldsymbol{y}_{\mathrm{CL}}) \cdot \dot{g}_{ij}(\boldsymbol{y}_{\mathrm{CL}}) \right). \tag{12}$$

**Definition 3** For a paired loop $y_i$-$u_j$, the relative expected gain (REG) is defined as the ratio between partial gain and expected gain

$$\bar{g}_{ij}(\boldsymbol{y}_{\mathrm{CL}}) = \dot{g}_{ij}(\boldsymbol{y}_{\mathrm{CL}}) / g_{ij}^{o},\ \boldsymbol{y}_{\mathrm{CL}} \subseteq \boldsymbol{y}^i. \tag{13}$$

Similarly, $2^{n-1}$ elements of REG form a set $\bar{\boldsymbol{A}}_i$:

$$\bar{\boldsymbol{A}}_i = \left\{ \bar{g}_{ij}(\boldsymbol{y}_{\mathrm{CL}}) \right\}. \tag{14}$$

Eq. (13) scales partial gain with its nominal value, and then the REG is independent of scaling.

### 3.3 Variance index as a measure of interaction

Now we can use $\bar{\boldsymbol{A}}_i$ to evaluate the interaction between loop $y_i$-$u_j$ and the other $(n-1)$ loops. $\bar{\boldsymbol{A}}_i$ contains $2^{n-1}$ scaled gains in all possible scenarios, with an expected value of unity. If REGs in $\bar{\boldsymbol{A}}_i$ are close to unity, it indicates that loop $y_i$-$u_j$ is not likely to be affected by the motions of other loops; if an REG is far away from unity, the partial gain experiences a significant deviation from its desired value.

Based on the above, the variance of $\bar{\boldsymbol{A}}_i$ (denoted as $\mathrm{var}(\bar{\boldsymbol{A}}_i)$) is used to represent the extent of deviation from expected unity. Clearly, $\mathrm{var}(\bar{\boldsymbol{A}}_i) \to 0$ ($i$=1, 2, ..., $n$) suggests less interaction and should be preferred. For the whole system, each loop has an REG set and then $n$ variances are obtained. An overall interaction index is needed and finally, a variance index (VI) is introduced.

**Definition 4** A VI is defined as the Euclidean norm of variance vector $\boldsymbol{V}$:

$$\mathrm{VI} \triangleq \|\boldsymbol{V}\|_2, \tag{15}$$

where $\boldsymbol{V}$ is the vector composed of $v_i$, and $v_i$ is the variance of $\bar{\boldsymbol{A}}_i$, i.e.,

$$\boldsymbol{V} \triangleq [v_i]^{\mathrm{T}} = [v_1 \cdots v_n]^{\mathrm{T}} = [\mathrm{var}(\bar{\boldsymbol{A}}_1) \cdots \mathrm{var}(\bar{\boldsymbol{A}}_n)]^{\mathrm{T}}, \tag{16}$$

and the Euclidean norm of $\boldsymbol{V}$ is defined as the square root of the inner product between $\boldsymbol{V}$ and itself:

$$\|\boldsymbol{V}\|_2 \triangleq \sqrt{\boldsymbol{V}^{\mathrm{T}}\boldsymbol{V}} = \sqrt{\sum_{i=1}^{n} v_i^2}.$$

Our aim is to select the pairing scheme with the least interaction. Therefore, we can calculate the VI for every pairing scheme $P^k$ and compare their VI values. The scheme with the minimum VI is selected. To explain our new measure, reconsider Example 1.

**Example 1** (continued) For the six candidate pairings given in Table 1, the REG values are calculated and plotted in Fig. 3, and values of variances and VI are given in Table 2. All scenarios are treated equally; i.e., the following equation is assumed:

$$\boldsymbol{M} = \boldsymbol{M}_{0.5} = [0.5 \cdots 0.5]^{\mathrm{T}}, \tag{17}$$

where $\boldsymbol{M}_\mu$ denotes that all elements in $\boldsymbol{M}$ are equal to $\mu$.

Fig. 3 intuitively gives information about how the REGs fluctuate around the unity line under different scenarios. Pairing schemes $P^2$ and $P^3$ behave relatively smoothly. The curves are located in a quite narrow region around the two sides of the unity line. Pairing schemes $P^5$ and $P^6$ experience intense fluctuations, and pairing schemes $P^1$ and $P^4$ perform moderately (Fig. 3). Note that with scheme $P^1$, which is the best candidate as suggested by the RGA-plus-NI rule, the curves start with spots near unity and end up with spots near unity. Thus, the RGA-plus-NI rule indicates that interaction is small when all loops are open and all other loops are closed. However, RGA overlooks the middle region of the curves where large interactions appear and results in selecting $P^1$ as the best choice. From Fig. 3 we predicted that $P^2$ and $P^3$ are likely to be good pairing schemes.

**Table 2 VI values of six candidate pairings for Example 1**

| Feasible pairing | Variance value | | | | VI |
|---|---|---|---|---|---|
| | $v_1$ | $v_2$ | $v_3$ | $v_4$ | |
| $P^1_{1432}$ | 0.9283 | 1.4401 | 2.0955 | 5.0314 | 5.7133 |
| $P^2_{3412}$ | 0.5378 | 0.6239 | 2.1030 | 2.1126 | 3.0926 |
| $P^3_{1234}$ | 0.9521 | 1.0845 | 0.0481 | 1.4610 | **2.0541** |
| $P^4_{3214}$ | 1.5274 | 2.8539 | 2.1253 | 3.1623 | 4.9995 |
| $P^5_{1342}$ | 9.9492 | 2.3751 | 6.9819 | 3.6917 | 12.9230 |
| $P^6_{4312}$ | 21.2995 | 3.5598 | 1.9490 | 7.4399 | 22.9236 |

Number in bold shows the smallest VI value, indicating $P^3$ is the best

**Remark 3** The $x$-axis numbering of the scenario in Fig. 3 is interpreted as follows: an integer $l$ ($0 \le l \le 2^{n-1}-1$) is transformed into an ($n-1$)-digit binary number, and the value in each digit represents the loop status of $\boldsymbol{y}^i$: 0 means open and 1 means closed. For example, consider a scenario number of 5 for $\bar{\boldsymbol{A}}_2$. The binary form of '5' is '101', and this implies that $y_1$ and $y_4$ are in perfect control, while the loop containing $y_3$ is open. Therefore, the curves start with the scenario that all loops are open and end up with the scenario that all other loops are closed.

Table 2 gives statistical information and VI values, which can be used as overall interaction measures. The VI of $P^3$ (2.0541) is minimum and thus $P^3$ is the best pairing, which is consistent with the results of Wolff and Skogestad (1990) and He *et al*.

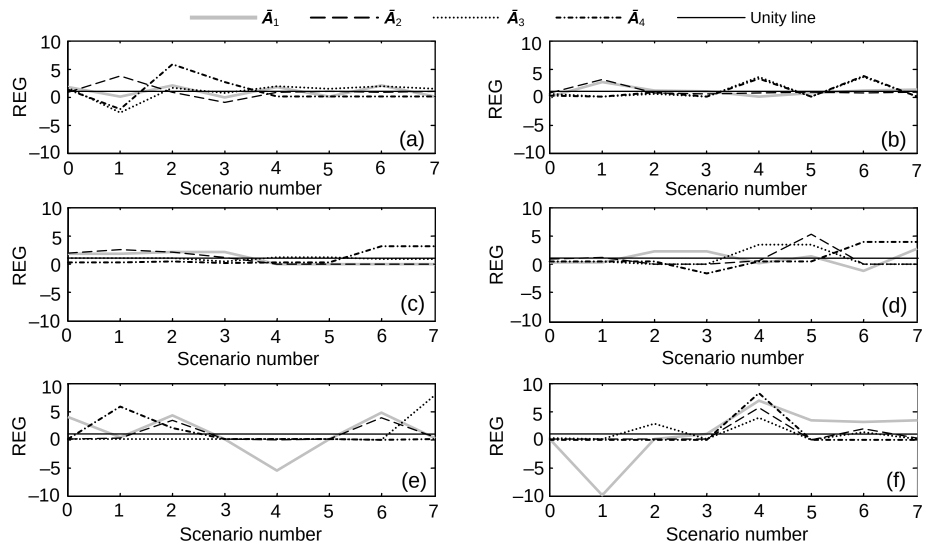


**Fig. 3 Relative expected gain values of six candidate pairings in different scenarios**
(a), (b), (c), (d), (e), and (f) correspond to schemes $P^1$, $P^2$, $P^3$, $P^4$, $P^5$, and $P^6$, respectively

(2004). Scheme $P^2$ gives a slightly higher VI (3.0926) and thus is the second best choice, and is the best in PRG analysis (Häggblom, 1997). Because PRG analysis highlights only one scenario $P^2$ experiences less interaction than $P^3$, but overlooks the overall interaction performances. $P^5$ and $P^6$ are the worst choices, as predicted.

Furthermore, check pairing schemes $P^1$, $P^3$, and $P^5$, $y_1$ are all paired with $u_1$, but their $v_1$ values (0.9283, 0.9521, 9.9492) differ distinctly. This confirms the conclusion that the interaction measure for one loop is dependent on how the other loops are paired. One can easily find more proofs in Table 2 (e.g., $v_4$ values for $P^1$, $P^2$, $P^5$, and $P^6$).

**Remark 4** The above results are derived on the precondition that Eq. (17) stands. Indeed, our interaction measure VI is dependent on the value of $\boldsymbol{M}$ (see Section 4 for more details).

## 4 Quantified integrity measure and a new pairing criterion

Next we consider the integrity problem. A necessary condition for loop $y_i$-$u_j$ to be integral stabilizable (Grosdidier *et al*., 1985) is

$$h_i(0) = g_{ij}(0)c_i(0) > 0, \tag{18}$$

where under a particular scenario, $g_{ij}(0)$ is equivalent to the partial gain $\dot{g}_{ij}(\boldsymbol{y}_{\mathrm{CL}})$. As a result, the sign of $h_i(0)$ is dependent on $\boldsymbol{G}_{\mathrm{CL}}$, as well as $c_i(0)$.

### 4.1 Expected integrity degree

For a specified $\boldsymbol{G}_{\mathrm{CL}}$ and considering only loop $y_i$-$u_j$, Eq. (18) becomes a necessary and sufficient condition for loop $y_i$-$u_j$ to be integral stabilizable. This is also the well-known result of classical feedback control theory. A problem emerges when $\boldsymbol{G}_{\mathrm{CL}}$ varies, which may lead to a sign change of $\dot{g}_{ij}(\boldsymbol{y}_{\mathrm{CL}})$. In this case, Eq. (18) no longer holds and instability occurs. Variation in $\boldsymbol{G}_{\mathrm{CL}}$ is caused by either operation mode switch or controller failure. A control system without integrity is usually undesired because the process is placed in potential danger.

In more detail, we divide the partial gain set $\boldsymbol{A}_i$ into two groups $\boldsymbol{A}_i^+$ and $\boldsymbol{A}_i^-$ according to their signs (zero element not included; it is critically unstable). If either $\boldsymbol{A}_i^+$ or $\boldsymbol{A}_i^-$ is empty, the controller can be easily designed to satisfy Eq. (18); otherwise, the issue becomes dependent on the sign of $c_i(0)$. The following three choices are suggested as references when choosing the sign of $c_i(0)$: (1) open loop gain; (2) expected gain; (3) $\boldsymbol{A}_i^+$ or $\boldsymbol{A}_i^-$, depending on which has higher probability.

Theoretically, the third reference should be taken to stabilize the majority of different scenarios. However, it is less laconic and will cost additional computation effort. We adopt the second reference for the following reasons:

1. All data are ready in $\bar{\boldsymbol{A}}_i$.
2. Zero expected gain rarely occurs.
3. The signs of the three references are identical for a strict integrity control system.
4. The signs of the three references are identical for a system without strict integrity but with a relatively well-paired control system.

The first two reasons facilitate calculation. The last two reasons indicate that differences between the three references appear only in poorly-paired control systems, which are not important.

Therefore, a negative element in $\bar{\boldsymbol{A}}_i$ implies that loop $y_i$-$u_j$ will go unstable when it is closed, since Eq. (18) is no longer satisfied. The closing of loop $y_i$-$u_j$ also drives the gains in other loops to change. However, only the stability problem in loop $y_i$-$u_j$ needs to be considered, because the system will be examined loop by loop. If a negative element occurs in $\bar{\boldsymbol{A}}_i$, the corresponding scenario that all loops in $\boldsymbol{G}^{\mathrm{OL}}$ are closed is recorded as unstable. The REG sets $\bar{\boldsymbol{A}}_i$ ($i$=1, 2, ..., $n$) will be checked in turn, and then all unstable scenarios are obtained. The other scenarios are stable.

**Remark 5** Unstable scenarios may overlap when different REG sets are examined; i.e., under such scenarios, more than one loop in $\boldsymbol{G}^{\mathrm{OL}}$ have a negative REG. They should not be counted repeatedly. A zero REG will be treated as unstable if encountered. In practice, it hardly ever occurs and has little impact on the final solution even when it does.

**Definition 5** Expected integrity degree (EID) is defined as the probability that the system is in stable scenarios. Mathematically, the EID can be formulated as

$$\mathrm{EID} \triangleq \sum_{S(\boldsymbol{y}^{\mathrm{OL}})\in \mathbf{SS}} \mu(\boldsymbol{y}^{\mathrm{OL}}) = 1 - \sum_{S(\boldsymbol{y}^{\mathrm{OL}})\in \mathbf{US}} \mu(\boldsymbol{y}^{\mathrm{OL}}),\ \boldsymbol{y}^{\mathrm{OL}} \subseteq \boldsymbol{y}, \tag{19}$$

where **SS** and **US** are the sets of stable and unstable scenarios respectively, $S(\boldsymbol{y}^{\mathrm{OL}})$ is the scenario that all loops of $\boldsymbol{y}^{\mathrm{OL}}$ are closed and $\boldsymbol{y}_{\mathrm{OL}}$ are uncontrolled, and $\mu(\boldsymbol{y}^{\mathrm{OL}})$ is the probability of $S(\boldsymbol{y}^{\mathrm{OL}})$.

During the steps of computing VI, all scenarios are enumerated, and therefore EID could be obtained easily. A DCLI control system has an EID value of 1. If Eq. (17) holds, the EID equals the ratio between the number of stable scenarios and $2^n$.

The EID is used as another index for pairing choice. Usually a DCLI control system is desired, and this could be generalized by the concept of EID: A pairing scheme with a high EID, i.e., EID→1, should be preferred. The significance of the EID comes from the following considerations:

1. There may be no DCLI control system for some plants, especially for large scale plants (Example 2). If only a fully decentralized control structure is allowed, we have to compromise and choose the most promising pairing scheme.

2. Being strictly, DCLI may be too conservative and prevent some good choices being made. A pairing scheme with an EID of less than 1 should also be considered, if the unstable scenarios occur with a low probability.

3. In practice, dynamic effects exist in a plant. Unstable scenarios will not cause the system to be out of control at once. If in transient phase the system is switched to a stable scenario, it could be recovered.

4. When unstable scenarios are obtained, they could be used as an operation guide. Unstable scenarios should be avoided as much as possible when the plant is running.

It is recommended that for a relatively low dimension system, a strict integrity property (i.e., EID=1) is required; for a high dimension system, this requirement could be relaxed.

### 4.2 New pairing criterion

Based on the above discussions, a new pairing criterion is proposed as follows: (1) All paired RGA elements are positive; (2) The value of VI should be small; (3) The value of EID should approach 1; (4) When rules (2) and (3) cannot be satisfied simultaneously, rule (3) holds higher priority in most cases.

Rule (1) is inherited from the RGA-plus-NI rule for the following two reasons: (1) In practice, open loop gain is intuitively used when choosing a controller; e.g., when a flow is controlled with a valve, an inverse-acting controller is always chosen. In a highly automated plant, all loops are supposed to be running automatically. Therefore, $g_{ij}$ and $\hat{g}_{ij}$ should have the same sign; i.e., the RGA element is positive. (2) Simple calculation of RGA eliminates poor pairing schemes efficiently, and the computation pain is greatly alleviated. The NI condition need not be checked because the stability problem has been included in rule (3).

**Example 1** (continued) Both VI and EID are dependent on probability vector $\boldsymbol{M}$. Five different values of $\boldsymbol{M}$ are taken to compare the results (Table 3).

First, note that only $P^2$ and $P^3$ have EID values of 1, implying that they are desired DCLI control configurations; the EID values of $P^1$ are all less than 1, which confirms the stability problem of $P^1$ noted earlier. Then compare the results under different $\boldsymbol{M}$ values. Interestingly, they all point to $P^3$ as the best pairing choice. This is in accordance with the result deduced above. However, we are not claiming that $P^3$ will invariably be the best choice. There are numerous possible values for $\boldsymbol{M}$. Table 3 supports the following conclusions concerning the loop-open probability vector $\boldsymbol{M}$:

1. The EID value of a DCLI system is 1, it is

**Table 3 Variance index and expected integrity degree results under different $\boldsymbol{M}$ values for Example 1**

| Feasible pairing | Variance index (VI) | | | | | Expected integrity degree (EID) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\boldsymbol{M}_{0.1}$ | $\boldsymbol{M}_{0.3}$ | $\boldsymbol{M}_{0.5}$ | $\boldsymbol{M}_{0.7}$ | $\boldsymbol{M}_{0.9}$ | $\boldsymbol{M}_{0.1}$ | $\boldsymbol{M}_{0.3}$ | $\boldsymbol{M}_{0.5}$ | $\boldsymbol{M}_{0.7}$ | $\boldsymbol{M}_{0.9}$ |
| $P^1_{1432}$ | 9.18 | 6.44 | 5.71 | 4.77 | 1.99 | 0.99 | 0.92 | 0.81 | 0.75 | 0.85 |
| $P^2_{3412}$ | 12.78 | 4.47 | 3.09 | 3.06 | 4.42 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| $P^3_{1234}$ | **8.13** | **3.24** | **2.05** | **2.08** | **0.68** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** |
| $P^4_{3214}$ | 13.39 | 5.42 | 5.00 | 5.05 | 2.46 | 0.99 | 0.92 | 0.81 | 0.75 | 0.85 |
| $P^5_{1342}$ | 47.63 | 12.81 | 12.92 | 35.75 | 655.30 | 0.99 | 0.92 | 0.81 | 0.92 | 0.85 |
| $P^6_{4312}$ | 136.00 | 17.25 | 22.92 | 1090.80 | 484.50 | 0.90 | 0.69 | 0.50 | 0.44 | 0.67 |

Numbers in bold show the smallest VI values and the highest EID values under different $\boldsymbol{M}$, indicating $P^3$ is always the best independent of $\boldsymbol{M}$; otherwise, it is dependent on $\boldsymbol{M}$. In addition, if no zero element occurs in $\boldsymbol{M}$, **SS** and **US** are independent of $\boldsymbol{M}$ (**US** in Example 1 can be found in Fig. 3, indicated by negative REGs).

2. VI is dependent on $\boldsymbol{M}$. This is reasonable because when a scenario is dominant, it should have high influence; when a particular scenario is unlikely to happen, its influence on the control system should be less emphasized. However, the final pairing result is not sensitive to $\boldsymbol{M}$, which is a desirable feature.

3. The $\boldsymbol{M}$ value is ambiguous in the design phase. It is suggested that if we have a priori knowledge of $\boldsymbol{M}$, then use it. Otherwise simply use $\boldsymbol{M}_{0.5}$ which still leads to a good pairing result. In fact, $\boldsymbol{M}_{0.5}$ represents the control system's internal physical property (all scenarios are treated equally); otherwise it represents the control system's performance under exogenous conditions.

The above conclusions show that a pairing decision can be made without being bothered by $\boldsymbol{M}$. The only indispensable information is the steady state gain matrix $\boldsymbol{G}$. Based on above discussions, the procedure of our proposed criterion can be summarized as follows:

Step 1: Obtain steady state gain matrix $\boldsymbol{G}$ and loop-open probability vector $\boldsymbol{M}$.

Step 2: Calculate the RGA by Eq. (3), and calculate the probability for $2^n$ different scenarios.

Step 3: Do primary screening according to rule (1), all pairing scheme candidates are obtained in $\{P^k\}$.

Step 4: Calculate REG set $\bar{\boldsymbol{A}}_i$ and variance $v_i$ for each loop in $P^k$ by Eqs. (6), (11)–(14) and (16). Record unstable scenarios by judging the signs of REG.

Step 5: Calculate VI by Eq. (15), obtain all unstable scenarios and calculate EID by Eq. (19).

Step 6: Go to Step 8 if calculation is done for all pairing schemes in $\{P^k\}$; otherwise go back to Step 4 to calculate the next pairing scheme candidate.

Step 7: Choose the best pairing scheme according to rules (2)–(4).

Step 8: End.

The algorithm description is also shown as a flowchart in Fig. 4.

**Remark 6** Steady state gain matrix $\boldsymbol{G}$ can be obtained by either setting $s$=0 in the transfer function matrix or by making a small perturbation around the steady state point. If the transfer function matrix has at least one integrator, the method provided by Arkun and Downs (1990) can be used to obtain an equivalent gain matrix.

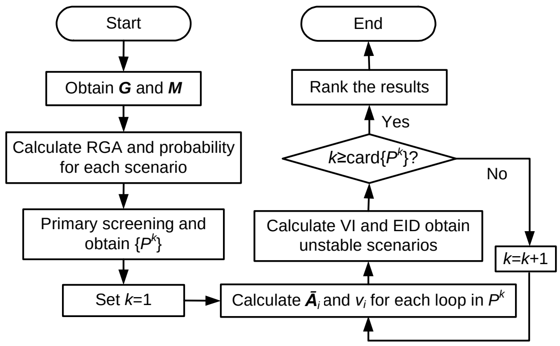


**Fig. 4 Flowchart of the proposed loop pairing criterion algorithm**

**Remark 7** If the scenarios are not independently distributed, then Eq. (11) no longer holds. In such a situation, we need to directly specify the probability of each scenario depending on the specific circumstances.

**Remark 8** The disadvantage of this method may be its computing complexity. However, control structure configuration is accomplished offline requiring computing only once and a computer program is written to perform this automatically. Developing an efficient algorithm is also one aspect of further work. Kariwala and Cao (2008) have applied a branch and bound method to pair variables based on RGA-number and $\mu$-interaction measure. They also noted that pairing selection shares a lot in common with the well-known traveling salesman problem.

## 5 Case study for a large scale system

In this section a large scale system is studied to test the effectiveness of our proposed criterion.

**Example 2** The Tennessee Eastman (TE) problem (Downs and Vogel, 1993) was studied by McAvoy and Ye (1994). The original TE problem had 12 manipulated variables and 41 measurements. In their Step 2 during design stage 2, McAvoy and Ye (1994) reduced the problem to a 7×7 system. Our example continues the task of control structure selection based on this 7×7 system. The manipulated variables, controlled variables, and the steady state gain matrix are shown in Table 4.

**Table 4 The 7×7 system for the TE problem given by McAvoy and Ye (1994)**

| | 1. A feed SP | 2. Purge SP | 3. Steam SP | 4. Rea Cl SP | 5. Sepa Cl SP | 6. Recycle valve | 7. Agit speed |
|---|---|---|---|---|---|---|---|
| 1. Rea feed flow | −8.2062 | −7.6820 | 0.0009 | −0.6652 | 0.2346 | −0.0360 | 0.1291 |
| 2. Rea temp | 6.4830 | 0.8968 | 0.0005 | 1.1106 | 0.0281 | 0.0141 | −0.2149 |
| 3. Rea pres | −3103.2488 | −965.7145 | 0.0477 | −31.4794 | 11.9680 | 7.7756 | 6.1135 |
| 4. Sepa temp | 67.8248 | 21.0053 | −0.0020 | 2.2502 | 0.2853 | −0.1954 | −0.4359 |
| 5. Stri temp | 46.0956 | 20.1120 | 0.0368 | 1.8663 | 0.4130 | −0.0380 | −0.3615 |
| 6. Recy flow | −11.2025 | −7.0837 | 0.0004 | −0.6510 | 0.2640 | −0.0187 | 0.1256 |
| **7.** Comp power | 126.7784 | −27.4335 | 0.0090 | −4.5126 | 2.5222 | 2.9658 | 0.8732 |

SP: setpoint. The SPs for inner loops are treated as manipulated variables for upper loops

The primary screening in Step 3 reduces the number of pairing candidates from $5.04\times10^3$ to 168. VI and EID values for the 168 candidates are plotted in Fig. 5, where $\boldsymbol{M}$ is set to be $\boldsymbol{M}_{0.5}$.

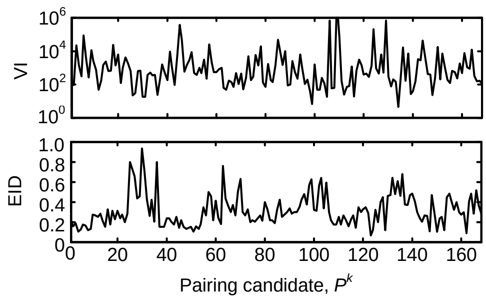


**Fig. 5 Variance index (VI) and expected integrity degree (EID) values of 168 pairing candidates for Example 2**

Surprisingly, even if pairing candidates are substantial, there are no DCLI control structures with an EID equal to 1, and only one candidate has an EID value greater than 0.8. This means that if we want the control structure to be a traditional DCLI, no feasible solution exists. The quantified integrity measure EID allows us to choose a relatively promising pairing scheme.

VI values also distribute diversely, ranging from around $10^1$ to $10^7$. These candidates are ranked according to our proposed criterion. Several typical pairing candidates are shown in Table 5. The best pairing scheme is $P^{30}$, with a relative small VI (17.2280, ranked as the 4th smallest) and the highest EID (0.9375). Note that $P^{134}$ has the smallest VI (4.3974) but a poor EID (0.6094), implying that 51 out of 128 scenarios are unstable.

Table 6 lists all unstable scenarios of pairing scheme $P^{30}$. It suggests that if $P^{30}$ is adopted, reliable actuators/sensors should be installed. It could also be used as an operation guide, as operators should prevent these scenarios occurring. $P^{30}$ was also selected by Kookos and Lygeros (1998), using an MILP programming algorithm based on the RIA interaction measure. Clearly, our proposed criterion offers much more information. Since plantwide control structure design is a systematic work, more aspects need to be investigated, e.g., disturbance rejecting ability, economic objective. However, as our solution stems from only a steady state gain matrix, it is very worthwhile.

**Remark 9** The above results are derived from the reduced 7×7 subsystem of the original TE problem. However, there are also interactions between this subsystem and other loops. Table 6 implicitly assumes that all other loops beyond this subsystem

**Table 5 Pairing candidates rank for Example 2**

| Rank | Pairing candidate | VI | EID |
|---|---|---|---|
| 1 | $P^{30}_{2715346}$ | 17.2280 | 0.9375 |
| 2 | $P^{36}_{2765341}$ | 23.4667 | 0.7969 |
| 3 | $P^{25}_{2713546}$ | 625.7494 | 0.7969 |
| ⋮ | ⋮ | ⋮ | ⋮ |
| *i* | $P^{134}_{6714325}$ | 4.3974 | 0.6094 |
| ⋮ | ⋮ | ⋮ | ⋮ |

**Table 6 Unstable scenarios of pairing scheme $P^{30}$ for Example 2**

| | $y_1$-$u_2$ | $y_2$-$u_7$ | $y_3$-$u_1$ | $y_4$-$u_5$ | $y_5$-$u_3$ | $y_6$-$u_4$ | $y_7$-$u_6$ |
|---|---|---|---|---|---|---|---|
| $\mathbf{US}_1$ | CL* | CL | | CL | | CL | |
| $\mathbf{US}_2$ | CL* | CL | | CL | CL | CL | |
| $\mathbf{US}_3$ | | CL* | | CL* | | CL* | |
| $\mathbf{US}_4$ | | CL* | | CL* | CL | CL* | |
| $\mathbf{US}_5$ | | CL | CL* | CL | | CL | |
| $\mathbf{US}_6$ | | CL | CL* | CL | CL | CL | |
| $\mathbf{US}_7$ | | CL | | CL | | CL | CL* |
| $\mathbf{US}_8$ | | CL | | CL | CL | CL | CL* |

CL denotes the corresponding loop is closed; otherwise it is open; CL* denotes the corresponding loop has a negative REG element remain unchanged. In this respect, Table 6 is not exactly correct. We need to apply our proposed criterion to the original TE problem if tight precision is required.

## 6 Conclusion

In this paper, a new loop pairing criterion was proposed for decentralized control structure configuration. It was stressed that interaction analysis is a pairing dependent issue. For each loop in a pairing candidate, the variations of gain under all possible scenarios were evaluated, and VI was defined as an overall interaction measure. Interaction analysis has essentially been extended to enable consideration from one arbitrary subsystem to another. EID was introduced as a quantified integrity measure. Both VI and EID were used as indices for pairing selection. Our proposed criterion overcame weaknesses of the traditional RGA-plus-NI rule, and an example was used to illustrate this point.

An additional large scale example was used to show the effectiveness of our proposed criterion. It was also pointed out that a plantwide problem should also consider other aspects, but our proposed criterion offers a satisfactory solution given limited information. In fact, in the design phase of plantwide control structure selection, our proposed criterion could also be used as an assessment of former steps. If no acceptable pairing solution exists, a mistake may probably have occurred in the first place; e.g., a poor choice of manipulated or controlled variables was made, or a decentralized control structure was inappropriate.

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