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Power control for two-way amplify-and-forward relaying over Rayleigh fading channels*

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Abstract: We propose two novel power control policies for a two-way amplify-and-forward (AF) relaying system, in which each node (two sources and one relay) is assumed to operate under both minimum and peak power constraints. Through the exploitation of instantaneous channel gains, the first policy can maximize the sum rate of the system. However, the instantaneous channel gains may be unavailable in a rapid time-varying system, where the first policy is inoperable. Consequently, a robust power control policy which requires only mean channel gains is proposed to maximize the upper bound of the average sum rate, and the properties of this policy are investigated. Simulation results show that, by comparison with the policy in which all the nodes use their peak transmit power, the proposed power control policies can provide considerable system performance improvement. Furthermore, the performance difference between the two proposed policies is negligible when the relay is close to one source.

Key words: Amplify-and-forward (AF) relaying, Power control, Two-way relay channel, Bidirectional relaying, Cooperative communication
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1 Introduction

Relay-assisted transmission has been shown to be a practical technique to extend the coverage of wireless communication systems (Pabst *et al*., 2004; Yang *et al*., 2009). A popular protocol is two-hop amplify-and-forward (AF) relaying (Laneman *et al*., 2004), which can be described in two phases. In the first phase, the source sends its message to a half-duplex relay. The relay amplifies the received message and forwards it to the destination in the second phase. Since the half-duplex relay cannot receive and transmit signals simultaneously, two channels should be used for the transmission of one information symbol. As a result, there is a pre-log factor 1/2 in corresponding expressions for the achievable spectral efficiency (Laneman *et al*., 2004). One way to avoid the pre-log factor 1/2 is to use the full-duplex relay, which can transmit and receive signals at the same time and frequency (Cover and El Gamal, 1979), but the large difference in power levels of the received and transmitted signals makes it difficult to implement this practically (Laneman, 2002). To recover the spectral efficiency loss, an efficient relaying protocol is proposed in which a synchronous bidirectional connection between two sources is established by a half-duplex AF relay (Rankov and Wittneben, 2007). This protocol is called two-way AF relaying (TWAR). The achievable sum rate of the TWAR system is investigated when given the instantaneous channel gains (Popovski and Yomo, 2007; Agustin *et al*., 2009). For a more efficient use of the power resource, power allocation for the TWAR system has been researched. Han *et al*. (2009) and Zhang *et al*. (2010) assumed that there is a total power

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constraint on the TWAR system and that the two sources in the system should use the same transmit power. To improve the upper bound of the average sum rate, a fixed power allocation ratio between the two sources and the relay is derived independently of the channel gains (Han *et al*., 2009). Zhang *et al*. (2010) proposed two power allocation strategies, both of which require only mean channel gains. The first strategy aims to maximize the upper bound of the average sum rate in high average SNR regions, and the second aims to achieve the trade-off of the outage probability between the two sources. Ho *et al*. (2008) introduced TWAR into the orthogonal frequency-division multiplexing (OFDM) system, which also has a total power constraint, and the power allocation maximizes the system capacity based on instantaneous channel gains.

It is found that in the TWAR system the rates of the two communication directions between the two sources constrain each other; that is to say, increasing the transmit power of one source leads to the decrement of its received SNR. Therefore, we may not obtain the maximum sum rate when all the nodes use their peak transmit power. In consideration of this, this paper will study the power control problem of the TWAR system in which each node has both minimum and peak power constraints. To the best of our knowledge, there are no published results in this area. First, an optimal power control policy is proposed to maximize the sum rate. This policy, however, requires that each source in the system have knowledge of instantaneous channel gain information of all the channels before transmission, which may hinder the implementation of this policy in some rapid time-varying systems. Subsequently, a robust power control policy is proposed to maximize the upper bound of the average sum rate in high average SNR regions through the exploitation of mean channel gains. Moreover, properties of this policy are investigated when the two sources have the same peak transmit power constraint.

2 System model

Considering a two-way communication protocol between two sources S_1 and S_2 (Fig. 1), it is assumed that there is no direct path between S_1 and S_2 due to large path loss or shadowing, and a half-duplex relay R helps the two sources exchange message using the TWAR protocol.

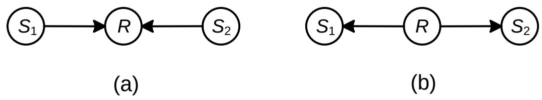

Fig. 1 System model for a two-way communication protocol between two sources
(a) MAC phase; (b) Broadcast phase

In the first (MAC) phase, S_1 and S_2 transmit their messages to R simultaneously, by assuming that all the channels are in flat-fading. The message received at R is given by

$$y_R = h_1 x_1 + h_2 x_2 + n_R, \tag{1}$$

where h_i denotes the complex channel gain between S_i and R, and x_i denotes the transmit message from S_i with the transmit power P_{S_i}, i=1, 2. n_R denotes the complex additive white Gaussian noise (AWGN) with noise power N_0.

In the second (broadcast) phase, y_R is amplified with a factor α and broadcasted to the two sources by the relay. Let P_R denote the transmit power of the relay. To meet the transmit power constraint, α is chosen as

$$\alpha = \sqrt{\frac{P_R}{|h_1|^2 P_{S_1} + |h_2|^2 P_{S_2} + N_0}}. \tag{2}$$

The channel gains are assumed to be constant during the two phases, so the messages received at S_1 and S_2 in the second phase are respectively given by

$$y_{S_1} = h_1(\alpha y_R) + n_{S_1}, \tag{3}$$

$$y_{S_2} = h_2(\alpha y_R) + n_{S_2}, \tag{4}$$

where n_{S_1} and n_{S_2} denote the complex AWGN with the same noise power N_0 at S_1 and S_2, respectively. We presume perfect knowledge of the corresponding channel gains at nodes S_1 and S_2. Since S_1 and S_2 know their own transmitted messages, they can subtract the back-propagating self-interference in Eqs. (3) and (4) before decoding (Rankov and Wittneben, 2007). Then we have

$$\tilde{y}_{S_1} = \alpha h_1 h_2 x_2 + \alpha h_1 n_R + n_{S_1}, \tag{5}$$

$$\tilde{y}_{S_2} = \alpha h_1 h_2 x_1 + \alpha h_2 n_R + n_{S_2}. \tag{6}$$

According to Eqs. (2), (5), and (6), the received signal-to-noise ratio (SNR) at S_1 and S_2 can be respectively expressed as

$$\mathrm{SNR}_{S_1} = \frac{\gamma_0\gamma_2|h_1|^2|h_2|^2}{\gamma_0|h_1|^2+\gamma_1|h_1|^2+\gamma_2|h_2|^2+1}, \tag{7}$$

$$\mathrm{SNR}_{S_2} = \frac{\gamma_0\gamma_1|h_1|^2|h_2|^2}{\gamma_0|h_2|^2+\gamma_1|h_1|^2+\gamma_2|h_2|^2+1}, \tag{8}$$

where $\gamma_0 = P_R/N_0$, $\gamma_1 = P_{S_1}/N_0$, and $\gamma_2 = P_{S_2}/N_0$. Therefore, the sum rate of the TWAR system is

$$R = 0.5\log_2\left[(1+\mathrm{SNR}_{S_1})(1+\mathrm{SNR}_{S_2})\right]. \tag{9}$$

Inspecting Eqs. (7) and (8), it is found that increasing the transmit power of one source leads to the decrement of its received SNR; therefore, self-interference still exists in the system. To prevent excessive self-interference, each source should have a peak transmit power constraint. Meanwhile, a minimum transmit power constraint of each node is necessary to guarantee a minimum achievable rate for each message transmission direction. Thus, it is assumed that in the TWAR system each node has both minimum and peak power constraints, which can be expressed as

$$\gamma_{\min,n} \le \gamma_n \le \gamma_{\max,n}, \quad n = 0,\ 1,\ 2, \tag{10}$$

where $\gamma_{\min,n}$ and $\gamma_{\max,n}$ denote the lower and upper bounds of γ_n, respectively. Note that, due to the existing self-interference, we may not achieve the maximum sum rate when all the nodes use their peak transmit power.

3 Two power control policies

From the analysis of the TWAR system in Section 2, the performance of the system could be improved through controlling the transmit power of each node. Therefore, we propose two power control policies to maximize two metrics of interest, the sum rate and the upper bound of the average sum rate in high average SNR regions.

3.1 Policy 1: the optimal power control policy

In this subsection, our objective is to maximize the sum rate of the TWAR system by designing a suitable power control policy. For this purpose, we can search for the optimal power control policy $\boldsymbol{\gamma}^* = [\gamma_0^*, \gamma_1^*, \gamma_2^*]$ by approaching the optimization problem

$$\boldsymbol{\gamma}^* = \arg\max_{\boldsymbol{\gamma}\in\Omega^3} R(\boldsymbol{\gamma}), \tag{11}$$

where $\Omega^3=\{\boldsymbol{\gamma}|\gamma_{\min,n}\le\gamma_n\le\gamma_{\max,n},\ n=0,\ 1,\ 2\}$ is a closed and bounded set, and R: $\Omega^3\rightarrow\mathbb{R}$ is continuous.

Define

$$\begin{aligned} f(\gamma_0,\gamma_1,\gamma_2) &\triangleq (1+\mathrm{SNR}_{S_1})(1+\mathrm{SNR}_{S_2}) \\ &\triangleq \left(1+\frac{\gamma_0\gamma_2|h_1|^2|h_2|^2}{1+\gamma_0|h_1|^2+\gamma_1|h_1|^2+\gamma_2|h_2|^2}\right) \\ &\quad\cdot\left(1+\frac{\gamma_0\gamma_1|h_1|^2|h_2|^2}{1+\gamma_0|h_2|^2+\gamma_1|h_1|^2+\gamma_2|h_2|^2}\right). \end{aligned} \tag{12}$$

According to Eq. (9), the optimization problem in Eq. (11) is equivalent to

$$\boldsymbol{\gamma}^* = \arg\max_{\boldsymbol{\gamma}\in\Omega^3} f(\boldsymbol{\gamma}). \tag{13}$$

Before proceeding, we note the following two lemmas (Gjendemsjø *et al.*, 2008).

Lemma 1 In the optimal power control policy, the relay uses its peak transmit power.

Proof After some manipulation, Eq. (12) can be equivalently transformed as

$$\begin{aligned} f(\gamma_0,\gamma_1,\gamma_2) &= \left(1+\frac{\gamma_2|h_1|^2|h_2|^2}{|h_1|^2+\left(1+\gamma_1|h_1|^2+\gamma_2|h_2|^2\right)/\gamma_0}\right) \\ &\quad\cdot\left(1+\frac{\gamma_1|h_1|^2|h_2|^2}{|h_2|^2+\left(1+\gamma_1|h_1|^2+\gamma_2|h_2|^2\right)/\gamma_0}\right). \end{aligned} \tag{14}$$

According to Eq. (14), it is obvious that $f(\gamma_0, \gamma_1, \gamma_2)$ is an increasing function with respect to γ_0. Therefore, the relay should use its peak transmit power to obtain the maximum value of $f(\boldsymbol{\gamma})$.

Lemma 2 In the optimal power control policy, at least one of the two sources uses its peak transmit power.

Proof If $\gamma_1 < \gamma_{\max,1}$, $\gamma_2 < \gamma_{\max,2}$, there exists a factor β which is larger than one and also satisfies

$$\begin{cases} \beta\gamma_1 \le \gamma_{\max,1}, \\ \beta\gamma_2 \le \gamma_{\max,2}. \end{cases} \tag{15}$$

We can prove that

$$\begin{aligned} & f(\gamma_0, \beta\gamma_1, \beta\gamma_2) \\ & = \left(1 + \frac{\gamma_0\gamma_2 |h_1|^2 |h_2|^2}{\left(1+\gamma_0 |h_1|^2\right)/\beta + \left(\gamma_1 |h_1|^2 + \gamma_2 |h_2|^2\right)}\right) \\ & \cdot \left(1 + \frac{\gamma_0\gamma_1 |h_1|^2 |h_2|^2}{\left(1+\gamma_0 |h_2|^2\right)/\beta + \left(\gamma_1 |h_1|^2 + \gamma_2 |h_2|^2\right)}\right) \\ & > f(\gamma_0, \gamma_1, \gamma_2). \end{aligned} \tag{16}$$

Thus, we can increase $f(\gamma_0, \gamma_1, \gamma_2)$ by increasing γ_1 and γ_2 by a factor β, until one of the two components reaches its upper bound. Hence, the solution of Eq. (13) will have at least one of the two sources use its peak transmit power.

According to Lemmas 1 and 2, we have the following theorem, which can be used to find the optimal power control policy:

Theorem 1 The optimal power control policy $\boldsymbol{\gamma}^*$ can be found amongst the following alternatives:

1. External points on the boundaries of Ω^3: for $\gamma_0=\gamma_{\max,0}$ and $\gamma_1=\gamma_{\max,1}$, the γ_2 corresponding to

$$\frac{\partial f(\gamma_{\max,0}, \gamma_{\max,1}, \gamma_2)}{\partial \gamma_2} = 0, \tag{17}$$

or for $\gamma_0=\gamma_{\max,0}$ and $\gamma_2=\gamma_{\max,2}$, the γ_1 corresponding to

$$\frac{\partial f(\gamma_{\max,0}, \gamma_1, \gamma_{\max,2})}{\partial \gamma_1} = 0. \tag{18}$$

2. Corner points of Ω^3: $[\gamma_{\max,0}, \gamma_{\max,1}, \gamma_{\max,2}]$, $[\gamma_{\max,0}, \gamma_{\max,1}, \gamma_{\min,2}]$, or $[\gamma_{\max,0}, \gamma_{\min,1}, \gamma_{\max,2}]$.

Taking the first derivatives of $f(\gamma_{\max,0}, \gamma_{\max,1}, \gamma_2)$ and $f(\gamma_{\max,0}, \gamma_1, \gamma_{\max,2})$ with respect to γ_2 and γ_1, respectively, we find that Eqs. (17) and (18) are both quadratic equations, so the closed-form solutions of these two equations can be obtained using standard analysis. Due to space limitation, we do not include the closed-form solutions in this paper. Based on the above analysis, the optimal power control policy $\boldsymbol{\gamma}^*$ can be found by the following steps.

Step 1: Given the instantaneous channel gains, first each source calculates the solutions of Eqs. (17) and (18).

Step 2: Each source checks whether the solutions obtained in Step 1 satisfy their power constraints, and then determines a set $\Delta\Omega^3$ that contains all the alternatives according to Theorem 1.

Step 3: The optimal power allocation policy $\boldsymbol{\gamma}^*$ can be obtained by solving the following problem:

$$\boldsymbol{\gamma}^* = \arg\max_{\boldsymbol{\gamma}\in\Delta\Omega^3} f(\boldsymbol{\gamma}). \tag{19}$$

Note that the optimization problem in Eq. (19) can be solved using an exhaustive search algorithm.

3.2 Policy 2: a sub-optimal power control policy

In the optimal power control policy, each source requires instantaneous channel gain information of all the channels before transmission in order to determine its optimal transmit power. However, the instantaneous channel gains may be unavailable in some systems due to substantial feedback requirement, overhead, and delay. Considering this, we propose a robust power control policy that requires only mean channel gain information.

According to Eq. (9), the average sum rate of the TWAR system can be expressed as

$$\begin{aligned} E[R] = & 0.5E\left[\log_2(1+\mathrm{SNR}_{S_1})\right] \\ & + 0.5E\left[\log_2(1+\mathrm{SNR}_{S_2})\right], \end{aligned} \tag{20}$$

where $E[\cdot]$ denotes the average of the variable. We attempt to maximize the average sum rate instead of the sum rate. Therefore, the optimization problem can be formulated as

$$\max_{\boldsymbol{\gamma}\in\Omega^3} E[R(\boldsymbol{\gamma})]. \tag{21}$$

Unfortunately, it is very difficult to solve the above optimization problem, which is based on the average sum rate. As an alternative, we can reduce the problem in (21) by maximizing the upper bound of the average sum rate in high average SNR regions, which allows us to develop a power control policy that needs only mean channel gain information. Based on the above, we have

$$E[R]\le 0.5\log_2\left[(1+E[\mathrm{SNR}_{S_1}])(1+E[\mathrm{SNR}_{S_2}])\right]$$
$$\le 0.5\log_2\left[\left(1+\frac{\gamma_0\gamma_2 g_1 g_2}{\gamma_0 g_1+\gamma_1 g_1+\gamma_2 g_2+1}\right)\cdot\left(1+\frac{\gamma_0\gamma_1 g_1 g_2}{\gamma_0 g_2+\gamma_1 g_1+\gamma_2 g_2+1}\right)\right], \quad (22)$$

where $g_1=E[|h_1|^2]$ and $g_2=E[|h_2|^2]$. In Eq. (22), the first step makes use of the well-known Jensen inequality and the second step follows the computation of average SNR (Deng and Haimovich, 2005; Zhang *et al.*, 2010). Furthermore, making use of the high-SNR approximations, Eq. (22) can be approximately written as

$$E[R]\le 0.5\log_2\left(\frac{\gamma_0\gamma_2 g_1 g_2}{\gamma_0 g_1+\gamma_1 g_1+\gamma_2 g_2}\frac{\gamma_0\gamma_1 g_1 g_2}{\gamma_0 g_2+\gamma_1 g_1+\gamma_2 g_2}\right). \quad (23)$$

Define

$$J(\gamma_0,\gamma_1,\gamma_2)\triangleq\frac{\gamma_0\gamma_2 g_1 g_2}{\gamma_0 g_1+\gamma_1 g_1+\gamma_2 g_2}\frac{\gamma_0\gamma_1 g_1 g_2}{\gamma_0 g_2+\gamma_1 g_1+\gamma_2 g_2}. \quad (24)$$

Then, the objective in (21) is reduced to finding the power control policy $\boldsymbol{\gamma}^{**}=[\gamma_0^{**},\gamma_1^{**},\gamma_2^{**}]$ by

$$\boldsymbol{\gamma}^{**}=\arg\max_{\gamma\in\Omega^3} J(\gamma_0,\gamma_1,\gamma_2). \quad (25)$$

Considering Eq. (24), it is observed that $J(\gamma_0, \gamma_1, \gamma_2)$ is an increasing function with respect to γ_0, so the relay should use its peak transmit power to obtain the maximum value of $J(\gamma_0, \gamma_1, \gamma_2)$. Moreover, for any $\lambda>1$, we have

$$J(\gamma_0,\lambda\gamma_1,\lambda\gamma_2)=\frac{\gamma_0\gamma_2 g_1 g_2}{\gamma_0 g_1/\lambda+\gamma_1 g_1+\gamma_2 g_2}\frac{\gamma_0\gamma_1 g_1 g_2}{\gamma_0 g_2/\lambda+\gamma_1 g_1+\gamma_2 g_2}>J(\gamma_0,\gamma_1,\gamma_2). \quad (26)$$

Comparison of Eq. (26) with Eq. (16) shows that $J(\gamma_0, \gamma_1, \gamma_2)$ has the same property as $f(\gamma_0, \gamma_1, \gamma_2)$. As a result, similar to Eq. (13), the solution of Eq. (25) should also have at least one of the two sources use its peak transmit power. Based on the above analysis, we can conclude that Lemmas 1 and 2 should also be satisfied by the sub-optimal power control policy, and the solution of Eq. (25) can be obtained by solving

$$\max\left\{J(\gamma_{\max,0},\gamma_{\max,1},\gamma_2),J(\gamma_{\max,0},\gamma_1,\gamma_{\max,2})\right\}$$
$$\text{s.t.}\quad \gamma_{\min,1}\le\gamma_1\le\gamma_{\max,1},\ \gamma_{\min,2}\le\gamma_2\le\gamma_{\max,2}. \quad (27)$$

Let $\tilde{\gamma}_1$ and $\tilde{\gamma}_2$ denote the maximum numbers of points of $J(\gamma_{\max,0}, \gamma_1, \gamma_{\max,2})$ and $J(\gamma_{\max,0}, \gamma_{\max,1}, \gamma_2)$ in their feasible regions, respectively. Taking the first derivative of $J(\gamma_{\max,0}, \gamma_{\max,1}, \gamma_2)$ with respect to γ_2, we have

$$\frac{\partial J(\gamma_{\max,0},\gamma_{\max,1},\gamma_2)}{\partial\gamma_2}=\frac{\gamma_{\max,0}^2\gamma_{\max,1}g_1^2g_2^2(AB-\gamma_2^2g_2^2)}{(A+\gamma_2g_2)^2(B+\gamma_2g_2)^2}, \quad (28)$$

where $A=\gamma_{\max,0}g_1+\gamma_{\max,1}g_1$ and $B=\gamma_{\max,0}g_2+\gamma_{\max,1}g_1$. Because γ_2 should be positive, the solution of $\frac{\partial J(\gamma_{\max,0},\gamma_{\max,1},\gamma_2)}{\partial\gamma_2}=0$ is

$$\bar{\gamma}_2=\sqrt{\frac{g_1}{g_2}(\gamma_{\max,0}+\gamma_{\max,1})\left(\gamma_{\max,0}+\gamma_{\max,1}\frac{g_1}{g_2}\right)}. \quad (29)$$

According to Eq. (28), $J(\gamma_{\max,0}, \gamma_{\max,1}, \gamma_2)$ is an increasing function of γ_2 in $(0, \bar{\gamma}_2]$, and a decreasing function in $[\bar{\gamma}_2, +\infty)$, so $\tilde{\gamma}_2$ can be obtained through analyzing the following three cases.

Case 1: if $\gamma_{\min,2}<\bar{\gamma}_2<\gamma_{\max,2}$, then $\tilde{\gamma}_2=\bar{\gamma}_2$.

Case 2: if $\bar{\gamma}_2\ge\gamma_{\max,2}$, then $\tilde{\gamma}_2=\gamma_{\max,2}$.

Case 3: if $\bar{\gamma}_2\le\gamma_{\min,2}$, then $\tilde{\gamma}_2=\gamma_{\min,2}$.

Due to the symmetry of the system model, the solution of $\frac{\partial J(\gamma_{\max,0},\gamma_1,\gamma_{\max,2})}{\partial\gamma_1}=0$ is

$$\bar{\gamma}_1=\sqrt{\frac{g_2}{g_1}(\gamma_{\max,0}+\gamma_{\max,2})\left(\gamma_{\max,0}+\gamma_{\max,2}\frac{g_2}{g_1}\right)}. \quad (30)$$

Similarly, $\tilde{\gamma}_1$ can be obtained using the same method as $\tilde{\gamma}_2$. Then, the sub-optimal power control policy can be obtained by

$$\boldsymbol{\gamma}^{**}=\arg\max\left\{J(\gamma_{\max,0},\gamma_{\max,1},\tilde{\gamma}_2),J(\gamma_{\max,0},\tilde{\gamma}_1,\gamma_{\max,2})\right\}. \quad (31)$$

Note that when $\bar{\gamma}_2\geq\gamma_{\max,2}$ or $\bar{\gamma}_1\geq\gamma_{\max,1}$, there exists the possibility that all the nodes choose to use their peak transmit power by adopting the proposed sub-optimal power control policy.

3.3 Two properties of the sub-optimal power control policy

Let Policy 3 denote the policy in which all the nodes use their peak transmit power, i.e., $\gamma_n=\gamma_{\max,n}$, n=0, 1, 2. From the above analysis, it is found that Policy 2 may degenerate to Policy 3, when there is no system performance improvement by using Policy 2 compared with Policy 3. Define Policy 2 as effective if Policy 2 does not degenerate to Policy 3 and consider a special case

$$\gamma_{\max,1}=\gamma_{\max,2}=\gamma_{\max}. \quad (32)$$

Let μ denote $\gamma_{\max}/\gamma_{\max,0}$. Then Policy 2 has the following two properties.

Property 1 Policy 2 is effective when

$$\frac{g_1}{g_2}<\kappa \quad \text{or} \quad \frac{g_2}{g_1}<\kappa, \quad (33)$$

where $\kappa=-\frac{1}{2\mu}+\sqrt{\frac{1}{4\mu^2}+\frac{\mu}{1+\mu}}$ represents the upper bound of the effective region of Policy 2. It is obvious that a larger κ is expected for Policy 2 to be effectively implemented.

Proof For $g_1<g_2$, we know

$$\bar{\gamma}_1=\sqrt{\frac{g_2}{g_1}(\gamma_{\max,0}+\gamma_{\max})\left(\gamma_{\max,0}+\gamma_{\max}\frac{g_2}{g_1}\right)}>\gamma_{\max}. \quad (34)$$

Thus, S_1 will always use its peak transmit power. Policy 2 is effective only when

$$\bar{\gamma}_2=\sqrt{\frac{g_1}{g_2}(\gamma_{\max,0}+\gamma_{\max})\left(\gamma_{\max,0}+\gamma_{\max}\frac{g_1}{g_2}\right)}<\gamma_{\max}. \quad (35)$$

After some manipulation, Eq. (35) can be written as

$$\varphi\left(\frac{g_1}{g_2}\right)\triangleq(\mu^2+\mu)\frac{g_1^2}{g_2^2}+(1+\mu)\frac{g_1}{g_2}-\mu^2<0. \quad (36)$$

Noticing $\mu^2+\mu>0$, $\varphi(0)<0$, and $\varphi(1)>0$, it is clear that the solution of Eq. (36) is

$$\frac{g_1}{g_2}<\frac{-1}{2\mu}+\sqrt{\frac{1}{4\mu^2}+\frac{\mu}{1+\mu}}. \quad (37)$$

For $g_1>g_2$, S_2 will always use its peak transmit power. Due to the symmetry of the system, Policy 2 is effective if

$$\frac{g_2}{g_1}<\frac{-1}{2\mu}+\sqrt{\frac{1}{4\mu^2}+\frac{\mu}{1+\mu}}. \quad (38)$$

For $g_1=g_2$, Policy 2 will definitely degenerate to Policy 3.

Property 2 The effective region of Policy 2 becomes larger with the increase of μ.

Proof Taking the first derivative of κ with respect to μ, we have

$$\frac{\mathrm{d}\kappa}{\mathrm{d}\mu}=\frac{\sqrt{4\mu^3+\mu+1}-\sqrt{\mu+1}}{2\mu^2\sqrt{4\mu^3+\mu+1}}+\frac{\sqrt{4\mu^3+4\mu^2}}{4(\mu^2+\mu)^2\sqrt{4\mu^3+\mu+1}}>0. \quad (39)$$

It is obvious that κ is an increasing function with respect to μ; consequently, the effective region of Policy 2 will be enlarged when μ increases. It is known that μ represents the peak transmit power ratio between the source and the relay, so the effective region of Policy 2 could be enlarged by increasing peak transmit power of the source, or reducing peak transmit power of the relay.

4 Simulation results

For illustrative purposes, we consider a linear topology, where the link S_1–R–S_2 forms a straight line. The distance between the two sources is normalized to 1, and the normalized distance between S_1 and R is d. The channel gain between each source and the relay introduces propagation path loss and Rayleigh fading, assuming the path loss exponent is 3. Thus,

$$\frac{g_1}{g_2}=\left(\frac{d}{1-d}\right)^{-3}. \qquad (40)$$

Fig. 2 shows the average sum rate versus d for the TWAR system, in which γ_1 and γ_2 are both bounded by [6, 12] dB, and γ_{max}=9 dB. The proposed power control policies are compared to Policy 3. It is observed that Policy 1 always outperforms Policy 3, and that the performance gain between the two policies grows larger when the relay moves towards one source from the center. Fig. 2 also illustrates that Policy 2 outperforms Policy 3 in its effective region, and that the performance difference between the two policies also grows larger when the relay moves towards one source from the center. Note that when the relay is close to one source (d<0.25 or d>0.75), Policy 2 can provide a similar performance to Policy 1.

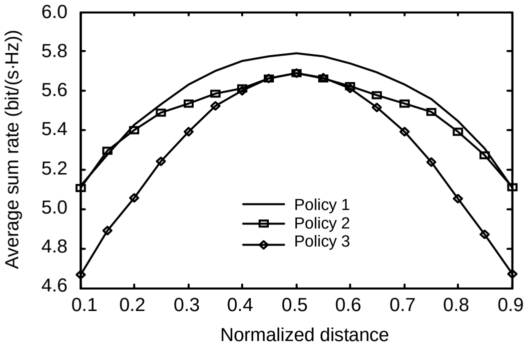

Fig. 2 Average sum rate versus the normalized distance between two sources when these two sources have the same transmit power constraint

Fig. 3 shows the average sum rate versus d when γ_1 and γ_2 are bounded by [6, 12] dB and [6, 10] dB respectively, and $\gamma_{max,0}$=9 dB. The proposed power control policies are more efficient when the relay is close to S_1 than when the relay is close to S_2. The reason for this behavior is that, in Policy 3, the influence of self-interference on the system performance is larger when the relay is close to S_1, which has a higher peak transmit power. Thus, more system performance improvement can be obtained through power control.

Fig. 4 illustrates the power saving property of Policy 2 in comparison with Policy 3. It is assumed that γ_1 and γ_2 are both bounded by [6, 12] dB, as in Fig. 2. Simulation results illustrate that when Policy 2 is effective (d<0.418 for μ=1 and d<0.458 for μ=2), S_1 can expect some power saving by adopting Policy 2. In particular, when the relay is close to S_1, S_1 will choose to use its minimum transmit power due to serious self-interference, in which case 75% of the transmit power can be saved for S_1. Moreover, according to Eq. (40) and the simulation results given in Fig. 4, we could calculate that the upper bound κ is equal to 0.3705 and 0.6034 for μ=1 and μ=2 respectively, which verifies Properties 1 and 2.

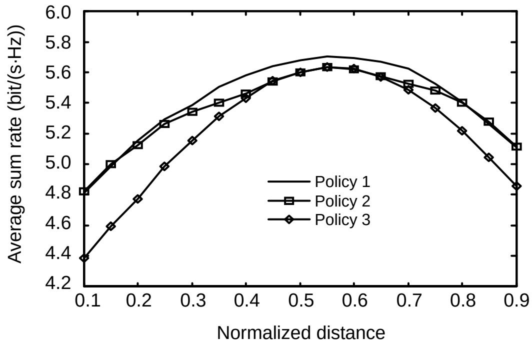

Fig. 3 Average sum rate versus the normalized distance between two sources when these two sources have different transmit power constraints

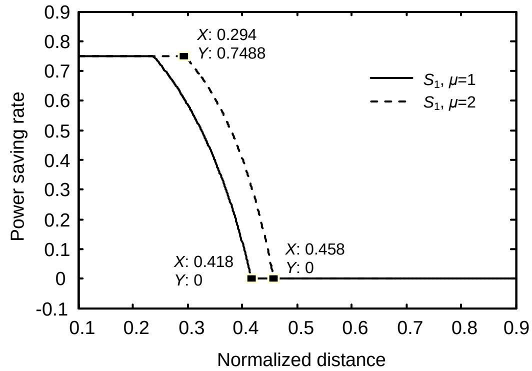

Fig. 4 Power saving rate versus the normalized distance between two sources

5 Conclusions

We propose two power control policies for the TWAR system in which each node has both minimum and peak power constraints. The maximum sum rate of the system can be achieved by adopting Policy 1, in which each source needs all the instantaneous channel gains before transmission. Compared with Policy 1, Policy 2 is a robust power control policy which is proposed to maximize the upper bound of the average sum rate in high average SNR regions and requires only mean channel gains. Numerical results show that the two proposed power control policies outperform the policy in which all the nodes use their peak transmit power, and that Policy 2 can provide similar performance to Policy 1 when the relay is close to one source. Using the two proposed power control policies, we can improve the average sum rate of the TWAR system and save some power resource.

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