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A construction of inter-group complementary codes with flexible ZCZ length*

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Abstract: A general construction of inter-group complementary (IGC) codes is proposed based on perfect complementary (PC) codes, interleaving operation, and the orthogonal matrix. The correlation properties of the newly constructed IGC codes can be described as follows: (1) the autocorrelation sidelobes of the codes are zeros in the zero correlation zone (ZCZ); (2) the cross-correlation functions (CCFs) between any two different codes of the same group are zeros in the ZCZ; (3) the CCFs between any two codes of different groups are zeros everywhere. The key point of this construction is that the ZCZ length of the generated IGC codes can be chosen flexibly. It is well known that there is a limitation between the ZCZ length and the number of mates; that is, the smaller is the length of ZCZ, the more are the IGC codes that can be generated. Therefore, if we can choose the ZCZ length of the IGC codes flexibly according to the requirement of the system, more users can be accommodated in the system.

Key words: Z-complementary codes, Perfect complementary codes, Zero correlation zone
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1 Introduction

Many technologies are investigated for the future wireless personal communications such as multiple-input multiple-output (MIMO) and orthogonal frequency division multiplexing (OFDM). Code division multiple access (CDMA) technology is attractive because its powerful processing gain brings many advantages, such as a low frequency reuse factor and robust performance. However, the performance of a CDMA system will be limited by various interferences such as multipath interference (MI) and multiple access interference (MAI) resulting from multipath propagation and non-ideal synchronization. Both MI and MAI can be measured by the correlation properties of the spreading codes. Therefore, designing new kinds of spreading codes with good correlation properties is an important scheme to improve the performance of different kinds of CDMA systems including CDMA, OFDM-CDMA, and MIMO-CDMA systems.

Many researchers have paid attention to the construction and application of spreading codes to mitigate interferences. The complete complementary (CC) codes were first proposed by Golay (1961) and Turyn (1963), and further improved by Suehiro, etc. (Suehiro, 1982; Suehiro and Hatori, 1988). A multi-carrier CDMA system employing CC codes was proposed by Chen *et al.* (2001). It is shown that the proposed system outperforms that employing traditional codes due to an enhanced spectral efficiency, especially in some scenarios where a multipath problem and multiple users exist. Furthermore, OFDM

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technology can be applied to simplify the hardware in a CDMA system employing CC codes. Unfortunately, the set size of the CC codes is limited severely to L, which is the length of elementary code (Tseng and Bell, 2000).

A more general complementary code called perfect complementary (PC) code was proposed (Chen and Chiu, 2004; Chen *et al.*, 2006b) to accommodate more users because its set size is equal to the elementary codes' number instead of their length. However, the number of available PC codes is limited in practice because the elementary codes must be transmitted via independent sub-channels, which are limited by the system resources.

A generalized complementary code called Z-complementary code was proposed to accommodate more users by reducing the length of the zero correlation zone (ZCZ) (Fan *et al.*, 2007). In comparison with PC codes, Z-complementary codes have neither limitation on the length of elementary codes, nor limitation on the number of mates. The synchronization or quasi-synchronization is required, however, when Z-complementary codes are employed by multi-carrier CDMA systems.

Generalized pairwise complementary (GPC) code (Chen *et al.*, 2006a) was proposed based on CC codes, providing the following correlation properties: each code set is composed of two groups, and the CCFs between any two different codes of the same group have a ZCZ; the CCFs between any two codes of different groups are zeros everywhere. The number of elementary codes of a GPC code set limits the number of groups. Generalized pairwise Z-complementary (GPZ) code (Feng *et al.*, 2008) was proposed based on Z-complementary codes, providing the following correlation properties: the CCFs between any two codes have a ZCZ; due to the advantages of the Z-complementary code, GPZ code can accommodate more users in the system when the synchronization or quasi-synchronization is required.

An inter-group complementary (IGC) code (Li *et al.*, 2008) was proposed based on PC codes. The IGC code set can be separated into several groups. The CCFs between any two different codes of the same group have a ZCZ. The CCFs between any two codes of different groups are zeros everywhere. A code assignment algorithm was also proposed (Li *et al.*, 2008) to show that the CDMA systems employing the IGC codes (IGC-CDMA) outperform the traditional CDMA systems employing conventional codes. Moreover, the IGC code sets can work well not only in synchronous mode but also in asynchronous mode. However, the ZCZ length of IGC codes (Li *et al.*, 2008) is fixed to the length of the elementary codes of the original PC codes, which limits the number of IGC codes.

As far as the authors are aware, most of the constructions of ZCZ sequence sets and Z-complementary codes are based on perfect sequences and PC codes, which cannot provide flexible choice of the ZCZ length due to the scarce length of perfect sequences and PC codes. A shift sequence set is constructed to generate ZCZ sequences with flexible ZCZ length (Zhou *et al.*, 2008), which reduces the construction of ZCZ sequences to the construction of shift sequences.

In this paper, a construction of IGC codes is proposed based on PC codes, interleaving operation, and the orthogonal matrix. The newly generated IGC codes have the following correlation properties: the CCFs between any two different codes of the same group have a ZCZ; the CCFs between any two codes of different groups are zeros everywhere. The key point of this method is that the length of ZCZ can be chosen freely according to the requirement of the system, which means that more codes can be generated when a short ZCZ is required.

2 Preliminaries

Let $\boldsymbol{a}=(a(0), a(1), \ldots, a(L-1))$ and $\boldsymbol{b}=(b(0), b(1), \ldots, b(L-1))$ denote two codes of length L with $a(i)=\pm1$ and $b(i)=\pm1$ $(0\le i<L)$, respectively. The periodic cross-correlation function (CCF) of $\boldsymbol{a}$ and $\boldsymbol{b}$ can be defined as

$$\boldsymbol{R}_{\boldsymbol{a},\boldsymbol{b}}(\tau)=\sum_{j=0}^{L-1}a(j)b(j+\tau),\ \ 0\le\tau<L, \tag{1}$$

where the addition is performed modulo L. When $\boldsymbol{a}=\boldsymbol{b}$, the above definition becomes a periodic auto-correlation function (ACF). A set of P binary codes $\{\boldsymbol{a}_0, \boldsymbol{a}_1, \ldots, \boldsymbol{a}_{P-1}\}$, each having length L, is called a PC code set if

$$\sum_{i=0}^{P-1}\boldsymbol{R}_{\boldsymbol{a}_i,\boldsymbol{a}_i}(\tau)=\begin{cases}LP, & \tau=0,\\ 0, & 1\le\tau<L.\end{cases} \tag{2}$$

Another set $\{\boldsymbol{b}_0, \boldsymbol{b}_1, \ldots, \boldsymbol{b}_{P-1}\}$ is called a mate of the PC code set $\{\boldsymbol{a}_0, \boldsymbol{a}_1, \ldots, \boldsymbol{a}_{P-1}\}$ if

$$\sum_{i=0}^{P-1} \boldsymbol{R}_{\boldsymbol{a}_i,\boldsymbol{b}_i}(\tau) = 0, \quad 0 \le \tau < L. \tag{3}$$

$\{\boldsymbol{a}_0, \boldsymbol{a}_1, \ldots, \boldsymbol{a}_{P-1}\}$ and $\{\boldsymbol{b}_0, \boldsymbol{b}_1, \ldots, \boldsymbol{b}_{P-1}\}$ are also called co-mates.

2.1 Definition of IGC codes

Given an IGC code set $I(K, P, N, W_{\min})$ (Li *et al.*, 2008) where K denotes the number of codes, P denotes the number of the elementary code, N denotes the length of the elementary code, and $W_{\min}$ denotes the ZCZ length, $K=PN/W_{\min}$. Those K codes can be divided into $G=P$ code groups I^g (g=0, 1, …, G–1), each group having $K/G=N/W_{\min}$ codes. The code set $I(K, P, N, W_{\min})$ has the following properties:

$$\boldsymbol{R}_{\boldsymbol{a},\boldsymbol{b}}(\tau) = \sum_{i=0}^{P-1} \boldsymbol{R}_{\boldsymbol{a}_i,\boldsymbol{b}_i}(\tau) = \begin{cases} PN, & \boldsymbol{a}=\boldsymbol{b},\ \tau=0, \\ 0, & \boldsymbol{a}=\boldsymbol{b},\ 0<|\tau|<W_{\min}, \\ 0, & \boldsymbol{a}\ne\boldsymbol{b},\ \boldsymbol{a},\boldsymbol{b}\in I^g,\ |\tau|<W_{\min}, \\ 0, & \boldsymbol{a}\in I^g, \boldsymbol{b}\in I^q, g\ne q,\ |\tau|<N, \\ \text{others}, & \text{otherwise}. \end{cases} \tag{4}$$

2.2 Definition of shift sequences

A set of shift sequences $E=\{\boldsymbol{e}_0, \boldsymbol{e}_1, \ldots, \boldsymbol{e}_i, \ldots, \boldsymbol{e}_{M-1}\}$, where $\boldsymbol{e}_i=(e_{i,0}, e_{i,1})$, $e_{i,j}$ is over $Z_L=\{0, 1, \ldots, L-1\}$, i=0, 1, …, M–1, j=0, 1, be obtained as the following (Zhou *et al.*, 2008). Given two positive integers L and Z where $2<Z<L$, we have two cases:

Case 1: Z is even. Let

$$M=\lfloor (L-2)/Z \rfloor,$$

where $\lfloor x \rfloor$ denotes the largest integer that is not larger than x. Then, $\boldsymbol{e}_i$ ($0\le i<M$) are obtained by

$$\boldsymbol{e}_i = (e_{i,0}, e_{i,1}) = \begin{cases} \left(\dfrac{Z}{2}i, L-1-\dfrac{Z}{2}(i+1)\right), & Z\,|\,(L-1), \\ \left(\dfrac{Z}{2}i, L-\dfrac{Z}{2}(i+1)\right), & \text{otherwise}, \end{cases} \tag{5}$$

where $x|y$ denotes y can be exactly divided by x. For example, let L=7 and Z=2. Then we have M=2, $\boldsymbol{e}_0$=(0, 5) and $\boldsymbol{e}_1$=(1, 4).

Case 2: Z is odd. Let

$$M=\lfloor (L-1)/Z \rfloor.$$

Then, $\boldsymbol{e}_i$ ($0\le i<M$) are obtained by

(i) $Z|L$

$$\boldsymbol{e}_i = (e_{i,0}, e_{i,1}) = \begin{cases} \left(\dfrac{Z}{2}i, L-1-\dfrac{Z-1}{2}-\dfrac{Z}{2}i\right), & i \text{ is even}, \\ \left(L-1-\dfrac{Z}{2}(i+1), \dfrac{iZ+1}{2}\right), & i \text{ is odd}. \end{cases} \tag{6}$$

(ii) $Z \nmid L$ (i.e., L cannot be exactly divided by Z)

$$\boldsymbol{e}_i = (e_{i,0}, e_{i,1}) = \begin{cases} \left(\dfrac{Z}{2}i, L-\dfrac{Z-1}{2}-\dfrac{Z}{2}i\right), & i \text{ is even}, \\ \left(L-\dfrac{Z}{2}(i+1), \dfrac{iZ+1}{2}\right), & i \text{ is odd}. \end{cases} \tag{7}$$

For example, Let L=7 and Z=3. Then we have M=2, $\boldsymbol{e}_0$=(0, 6) and $\boldsymbol{e}_1$=(4, 2).

For the shift sequences defined by Eqs. (5)–(7), for $0\le i, j<M$, the following inequalities hold (Zhou *et al.*, 2008):

$$\min_{\boldsymbol{e}_i,\boldsymbol{e}_j\in E;\boldsymbol{e}_i\ne\boldsymbol{e}_j} \{e_{i,0}-e_{j,0}, e_{i,1}-e_{j,1}\} > Z/2, \tag{8}$$

$$\min_{\boldsymbol{e}_i,\boldsymbol{e}_j\in E} \{e_{i,0}-e_{j,1}, e_{i,1}-e_{j,0}-1\} \ge (Z-1)/2, \tag{9}$$

$$e_{i,0}-e_{j,0} \ne e_{i,1}-e_{j,1} \quad \forall \boldsymbol{e}_i, \boldsymbol{e}_j \in E,\ \boldsymbol{e}_i \ne \boldsymbol{e}_j, \tag{10}$$

$$e_{i,0}-e_{j,1} \ne e_{i,1}-e_{j,0}-1 \quad \forall \boldsymbol{e}_i, \boldsymbol{e}_j \in E. \tag{11}$$

3 Construction of IGC codes

As stated in Section 1, the ZCZ length of the IGC codes (Li *et al.*, 2008) is fixed to the length of the elementary code of the original PC codes, which have many limitations in practical applications in CDMA systems due to the scarce length and the number of mates.

In this study, we propose a new construction to generate IGC codes with flexible ZCZ length based on PC codes, shift sequences, the orthogonal matrix, and interleaving operations. The detailed steps are presented as the following.

Step 1: Select an original PC code set, each having G groups $\boldsymbol{A}=\{\boldsymbol{A}^0, \boldsymbol{A}^1, \ldots, \boldsymbol{A}^{G-1}\}$, where $\boldsymbol{A}^g=\{\boldsymbol{a}_0^g, \boldsymbol{a}_1^g, \ldots, \boldsymbol{a}_{P-1}^g\}$ and $\boldsymbol{A}^k=\{\boldsymbol{a}_0^k, \boldsymbol{a}_1^k, \ldots, \boldsymbol{a}_{P-1}^k\}$ (g, k=0, 1, …, G−1 and $g\neq k$) are co-mates each with P elementary codes, and $\boldsymbol{a}_i^g=(a_i^g(0), a_i^g(1), \ldots, a_i^g(L-1))$ (i=0, 1, …, P−1) is one elementary code with length L. According to the requirement of the system, the ZCZ length of the newly generated IGC codes is preset to be Z ($0<Z<L$).

Step 2: According to L and Z assumed in Step 1, an appropriate shift sequence set E with set size M ($E=\{\boldsymbol{e}_0, \boldsymbol{e}_1, \ldots, \boldsymbol{e}_i, \ldots, \boldsymbol{e}_{M-1}\}$, $\boldsymbol{e}_i=(e_{i,0}, e_{i,1})$, i=0, 1, …, M−1) can be generated according to Zhou *et al.* (2008), as described in Section 2.

Step 3: Obtain interleaved sequence set $U^{g,1}$ from the elementary code $\boldsymbol{A}^g$ and the shift sequence E as follows:

$$U^{g,1}=\{\boldsymbol{U}_{i,j}^g:\ 0\le i<M, 0\le j<P\}, \tag{12}$$

$$\begin{aligned}\boldsymbol{U}_{i,j}^g &= (S^{e_{i,0}}(\boldsymbol{a}_j^g), S^{e_{i,1}}(\boldsymbol{a}_j^g))\\ &= (a_j^g(0+e_{i,0}),\ a_j^g(0+e_{i,1}),\ a_j^g(1+e_{i,0}), \ldots,\\ &\quad a_j^g(L-1+e_{i,0}),\ a_j^g(L-1+e_{i,1})),\end{aligned} \tag{13}$$

where S denotes the left cyclic shift operator, i.e., $S^1(\boldsymbol{a}_j^g)=(a_j^g(1),\ a_j^g(2),\ \ldots,\ a_j^g(L-1),\ a_j^g(0))$, and $a_j^g(k+e_{i,m})$ denotes the $((k+e_{i,m}) \bmod L)$th element of $\boldsymbol{a}_j^g$ for $0\le k<L$, $0\le i<M$, $0\le m<1$.

Step 4: Extend $U^{g,1}$ to obtain a larger set $U^g=U^{g,1}\cup U^{g,2}$, where the interleaved sequence set $U^{g,2}$ is obtained from the elementary code $\boldsymbol{A}^g$ and the shift sequence E as the following:

$$U^{g,2}=\{\boldsymbol{U}_{i+M,j}^g: 0\le i<M, 0\le j<P\}, \tag{14}$$

$$\begin{aligned}\boldsymbol{U}_{i+M,j}^g &= (S^{e_{i,0}}(\boldsymbol{a}_j^g), -S^{e_{i,1}}(\boldsymbol{a}_j^g))\\ &= (a_j^g(0+e_{i,0}), -a_j^g(0+e_{i,1}), a_j^g(1+e_{i,0}), \ldots,\\ &\quad a_j^g(L-1+e_{i,0}), -a_j^g(L-1+e_{i,1})).\end{aligned} \tag{15}$$

Based on the above steps, we obtain an IGC code set U which has G groups U^g (g=0, 1, …, G−1), each group having $2M$ codes $\boldsymbol{U}_i^g$ (i=0, 1, …, $2M$−1) and each code having P elementary codes $\boldsymbol{U}_{i,j}^g$ (j=0, 1, …, P−1) with length $N=2L$.

4 Correlation properties of generated IGC codes

Theorem 1 The newly generated IGC code set U has the following correlation properties:

1. The ACF sidelobes of each IGC code are zeros in the ZCZ with length of Z.

2. The CCFs between any two different codes of the same group are zeros in the ZCZ with length of Z.

3. The CCFs between any two codes of the different groups are zeros everywhere; i.e., their CCFs have a ZCZ with length of N.

Proof According to the theory of periodic correlations, the ACFs and CCFs are symmetric about the zero shift. Only the correlation functions with positive time shifts need to be discussed.

1. Without loss of generality, we take the ith code of the gth group $\boldsymbol{U}_i^g$ (which is generated by $\boldsymbol{A}^g$ (g=0, 1, …, G−1) and $\boldsymbol{e}_i$ (i=0, 1, …, M−1)) as an example. The value of the periodic ACFs at the shift $\tau=2\tau_1+\tau_2$ ($0\le\tau_1<L$, $0\le\tau_2<2$) can be calculated using two cases.

Case 1: if $\tau_2=0$, we have

(i) $\boldsymbol{U}_i^g\in U^{g,1}$

$$\begin{aligned}R_{\boldsymbol{U}_i^g,\boldsymbol{U}_i^g}(\tau) &= \sum_{j=0}^{P-1}R_{\boldsymbol{a}_j^g,\boldsymbol{a}_j^g}(\tau_1)+\sum_{j=0}^{P-1}R_{\boldsymbol{a}_j^g,\boldsymbol{a}_j^g}(\tau_1)\\ &= \begin{cases}2PL, & \tau_1=0,\\ 0, & \text{otherwise.}\end{cases}\end{aligned}$$

(ii) $\boldsymbol{U}_i^g\in U^{g,2}$

$$\begin{aligned}R_{\boldsymbol{U}_i^g,\boldsymbol{U}_i^g}(\tau) &= \sum_{j=0}^{P-1}R_{\boldsymbol{a}_j^g,\boldsymbol{a}_j^g}(\tau_1)+\sum_{j=0}^{P-1}R_{-\boldsymbol{a}_j^g,-\boldsymbol{a}_j^g}(\tau_1)\\ &= \begin{cases}2PL, & \tau_1=0,\\ 0, & \text{otherwise.}\end{cases}\end{aligned}$$

According to (i) and (ii), we have

$$R_{U_i^g,U_i^g}(\tau)=\begin{cases}2PL, & \tau_1=0,\\ 0, & \text{otherwise.}\end{cases} \tag{16}$$

Eq. (16) is obtained according to the ACFs properties of $\boldsymbol{A}^g$ (Chen *et al.*, 2006).

Case 2: if τ_2=1, we have

(i) $\boldsymbol{U}_i^g \in U^{g,1}$

$$\begin{aligned}R_{U_i^g,U_i^g}(\tau)&=\sum_{j=0}^{P-1}R_{a_j^g,a_j^g}(\tau_1-(e_{i,0}-e_{i,1}))\\&\quad+\sum_{j=0}^{P-1}R_{a_j^g,a_j^g}(\tau_1-(e_{i,1}-e_{i,0}-1))\\&=\begin{cases}0, & 0<|\tau_1|<(Z-1)/2,\\ \text{others}, & \text{otherwise.}\end{cases}\end{aligned}$$

(ii) $\boldsymbol{U}_i^g \in U^{g,2}$

$$\begin{aligned}R_{U_i^g,U_i^g}(\tau)&=\sum_{j=0}^{P-1}R_{a_j^g,-a_j^g}(\tau_1-(e_{i,0}-e_{i,1}))\\&\quad+\sum_{j=0}^{P-1}R_{-a_j^g,a_j^g}(\tau_1-(e_{i,1}-e_{i,0}-1))\\&=\begin{cases}0, & 0<|\tau_1|<(Z-1)/2,\\ \text{others}, & \text{otherwise.}\end{cases}\end{aligned}$$

According to (i) and (ii), we have

$$R_{U_i^g,U_i^g}(\tau)=\begin{cases}0, & 0<|\tau_1|<(Z-1)/2,\\ \text{others}, & \text{otherwise.}\end{cases} \tag{17}$$

Eq. (17) is obtained according to the ACFs properties of $\boldsymbol{A}^g$ (Chen *et al.*, 2006) and the properties of the shift sequence $\boldsymbol{e}_i$ (i=0, 1, …, M−1) (Zhou *et al.*, 2008).

According to Eqs. (16) and (17), we have

$$R_{U_i^g,U_i^g}(\tau)=\begin{cases}2PL, & \tau=0,\\ 0, & 0<|\tau|<Z,\\ \text{others}, & \text{otherwise.}\end{cases} \tag{18}$$

It is proved that the ACF of the generated IGC codes has a ZCZ with one-sided length of Z.

2. Without loss of generality, we take the ith and kth codes of the gth group ($\boldsymbol{U}_i^g$ and $\boldsymbol{U}_k^g$) as an example. Let $d_0=e_{i,0}-e_{k,0}$, $d_1=e_{i,1}-e_{k,1}$, $d_2=e_{i,0}-e_{k,1}$, and $d_3=e_{i,1}-e_{k,0}-1$. The periodic CCFs at shift $\tau=2\tau_1+\tau_2$ ($0\le\tau_1<L$, $0\le\tau_2<2$) can be calculated using two cases.

Case 1: if τ_2=0, we have

(i) $\boldsymbol{U}_i^g$, $\boldsymbol{U}_k^g \in U^{g,1}$

$$\begin{aligned}R_{U_i^g,U_k^g}(\tau)&=\sum_{j=0}^{P-1}R_{a_j^g,a_j^g}(\tau_1-d_0)+\sum_{j=0}^{P-1}R_{a_j^g,a_j^g}(\tau_1-d_1)\\&=\begin{cases}0, & |\tau_1|<Z/2,\\ \text{others}, & \text{otherwise.}\end{cases}\end{aligned}$$

(ii) $\boldsymbol{U}_i^g \in U^{g,1}$, $\boldsymbol{U}_k^g \in U^{g,2}$

$$\begin{aligned}R_{U_i^g,U_k^g}(\tau)&=\sum_{j=0}^{P-1}R_{a_j^g,a_j^g}(\tau_1-d_0)+\sum_{j=0}^{P-1}R_{a_j^g,-a_j^g}(\tau_1-d_1)\\&=\begin{cases}0, & |\tau_1|<Z/2,\\ \text{others}, & \text{otherwise.}\end{cases}\end{aligned}$$

(iii) $\boldsymbol{U}_i^g \in U^{g,2}$, $\boldsymbol{U}_k^g \in U^{g,1}$

$$\begin{aligned}R_{U_i^g,U_k^g}(\tau)&=\sum_{j=0}^{P-1}R_{a_j^g,a_j^g}(\tau_1-d_0)+\sum_{j=1}^{P-1}R_{-a_j^g,a_j^g}(\tau_1-d_1)\\&=\begin{cases}0, & |\tau_1|<Z/2,\\ \text{others}, & \text{otherwise.}\end{cases}\end{aligned}$$

(iv) $\boldsymbol{U}_i^g$, $\boldsymbol{U}_k^g \in U^{g,2}$

$$\begin{aligned}R_{U_i^g,U_k^g}(\tau)&=\sum_{j=0}^{P-1}R_{a_j^g,a_j^g}(\tau_1-d_0)+\sum_{j=0}^{P-1}R_{-a_j^g,-a_j^g}(\tau_1-d_1)\\&=\begin{cases}0, & |\tau_1|<Z/2,\\ \text{others}, & \text{otherwise.}\end{cases}\end{aligned}$$

According to (i), (ii), (iii), and (iv), we have

$$R_{U_i^g,U_k^g}(\tau)=\begin{cases}0, & |\tau_1|<Z/2,\\ \text{others}, & \text{otherwise.}\end{cases} \tag{19}$$

Case 2: if τ_2=1, we have

(i) $\boldsymbol{U}_i^g$, $\boldsymbol{U}_k^g \in U^{g,1}$

$$R_{U_i^g,U_k^g}(\tau)=\sum_{j=0}^{P-1}R_{a_j^g,a_j^g}(\tau_1-d_2)+\sum_{j=0}^{P-1}R_{a_j^g,a_j^g}(\tau_1-d_3)=\begin{cases}0, & |\tau_1|<(Z-1)/2,\\ \text{others}, & \text{otherwise.}\end{cases}$$

(ii) $U_i^g \in U^{g,1}$, $U_k^g \in U^{g,2}$

$$R_{U_i^g,U_k^g}(\tau)=\sum_{j=0}^{P-1}R_{a_j^g,-a_j^g}(\tau_1-d_2)+\sum_{j=0}^{P-1}R_{a_j^g,a_j^g}(\tau_1-d_3)=\begin{cases}0, & |\tau_1|<(Z-1)/2,\\ \text{others}, & \text{otherwise.}\end{cases}$$

(iii) $U_i^g \in U^{g,2}$, $U_k^g \in U^{g,1}$

$$R_{U_i^g,U_k^g}(\tau)=\sum_{j=0}^{P-1}R_{a_j^g,a_j^g}(\tau_1-d_2)+\sum_{j=0}^{P-1}R_{-a_j^g,a_j^g}(\tau_1-d_3)=\begin{cases}0, & |\tau_1|<(Z-1)/2,\\ \text{others}, & \text{otherwise.}\end{cases}$$

(iv) U_i^g, $U_k^g \in U^{g,2}$

$$R_{U_i^g,U_k^g}(\tau)=\sum_{j=0}^{P-1}R_{a_j^g,-a_j^g}(\tau_1-d_2)+\sum_{j=0}^{P-1}R_{-a_j^g,a_j^g}(\tau_1-d_3)=\begin{cases}0, & |\tau_1|<(Z-1)/2,\\ \text{others}, & \text{otherwise.}\end{cases}$$

According to (i), (ii), (iii), and (iv), we have

$$R_{U_i^g,U_k^g}(\tau)=\begin{cases}0, & |\tau_1|<(Z-1)/2,\\ \text{others}, & \text{otherwise.}\end{cases} \tag{20}$$

Eqs. (19) and (20) are obtained according to the ideal ACF property of $\boldsymbol{A}^g$ (g=0, 1, …, G−1) and the properties of $\boldsymbol{e}_i$ and $\boldsymbol{e}_k$ (i, k=0, 1, …, M−1; $i\neq k$). According to Eqs. (19) and (20), we have

$$R_{U_i^g,U_k^g}(\tau)=\begin{cases}0, & |\tau|<Z,\\ \text{others}, & \text{otherwise.}\end{cases} \tag{21}$$

3. Without loss of generality, we take the ith code of the gth group and the kth code of the qth group (U_i^g and U_k^q) as an example. Let $d_0=e_{i,0}-e_{k,0}$, $d_1=e_{i,1}-e_{k,1}$, $d_2=e_{i,0}-e_{k,1}$, and $d_3=e_{i,1}-e_{k,0}-1$. The periodic CCFs at shift $\tau=2\tau_1+\tau_2$ ($0\le\tau_1<L$, $0\le\tau_2<2$) can be calculated using two cases.

Case 1: if $\tau_2=0$, we have

(i) $U_i^g \in U^{g,1}$, $U_k^q \in U^{q,1}$

$$R_{U_i^g,U_k^q}(\tau)=\sum_{j=0}^{P-1}R_{a_j^g,a_j^q}(\tau_1-d_0)+\sum_{j=0}^{P-1}R_{a_j^g,a_j^q}(\tau_1-d_1)=0.$$

(ii) $U_i^g \in U^{g,1}$, $U_k^q \in U^{q,2}$

$$R_{U_i^g,U_k^q}(\tau)=\sum_{j=0}^{P-1}R_{a_j^g,a_j^q}(\tau_1-d_0)+\sum_{j=0}^{P-1}R_{a_j^g,-a_j^q}(\tau_1-d_1)=0.$$

(iii) $U_i^g \in U^{g,2}$, $U_k^q \in U^{q,1}$

$$R_{U_i^g,U_k^q}(\tau)=\sum_{j=0}^{P-1}R_{a_j^g,a_j^q}(\tau_1-d_0)+\sum_{j=0}^{P-1}R_{-a_j^g,a_j^q}(\tau_1-d_1)=0.$$

(iv) $U_i^g \in U^{g,2}$, $U_k^q \in U^{q,2}$

$$R_{U_i^g,U_k^q}(\tau)=\sum_{j=0}^{P-1}R_{a_j^g,a_j^q}(\tau_1-d_0)+\sum_{j=0}^{P-1}R_{-a_j^g,-a_j^q}(\tau_1-d_1)=0.$$

According to (i), (ii), (iii), and (iv), we have

$$R_{U_i^g,U_k^q}(\tau)=0. \tag{22}$$

Case 2: if $\tau_2=1$, we have

(i) $U_i^g \in U^{g,1}$, $U_k^q \in U^{q,1}$

$$R_{U_i^g,U_k^q}(\tau)=\sum_{j=1}^{P-1}R_{a_j^g,a_j^q}(\tau_1-d_2)+\sum_{j=1}^{P-1}R_{a_j^g,a_j^q}(\tau_1-d_3)=0.$$

(ii) $U_i^g \in U^{g,1}$, $U_k^q \in U^{q,2}$

$$R_{U_i^g,U_k^q}(\tau)=\sum_{j=1}^{P-1}R_{a_j^g,-a_j^q}(\tau_1-d_2)+\sum_{j=1}^{P-1}R_{a_j^g,a_j^q}(\tau_1-d_3)=0.$$

(iii) $U_i^g \in U^{g,2}$, $U_k^q \in U^{q,1}$

$$R_{U_i^g,U_k^q}(\tau)=\sum_{j=1}^{P-1}R_{a_j^g,a_j^q}(\tau_1-d_2)+\sum_{j=1}^{P-1}R_{-a_j^g,a_j^q}(\tau_1-d_3)=0.$$

(iv) $\boldsymbol{U}_i^g \in U^{g,2}$, $\boldsymbol{U}_k^q \in U^{q,2}$

$$R_{U_i^g,U_k^q}(\tau)=\sum_{j=1}^{P-1}R_{a_j^g,-a_j^q}(\tau_1-d_2)+\sum_{j=1}^{P-1}R_{-a_j^g,a_j^q}(\tau_1-d_3)=0.$$

According to (i), (ii), (iii), and (iv), we have

$$R_{U_i^g,U_k^q}(\tau)=0. \tag{23}$$

Eqs. (22) and (23) are derived from the ideal CCF property of $\boldsymbol{A}^g$ and $\boldsymbol{A}^q$ (g, q=0, 1, …, G–1; $g\neq q$), and the properties of $\boldsymbol{e}_i$ and $\boldsymbol{e}_k$ (i, k=0, 1, …, M–1). According to Eqs. (22) and (23), we have

$$R_{U_i^g,U_k^q}(\tau)=0,\quad 0\le|\tau|<N. \tag{24}$$

Based on Eqs. (18), (21), and (24), Theorem 1 is proved.

5 Discussion on the size of IGC codes

Given an original PC code set with G codes who are co-mates, each code having P elementary codes with length L and a shift sequence set with M sequences according to the predefined parameter Z, the newly generated IGC codes have $2MG$ codes; these codes can be divided into G groups, with each group having $2M$ codes and each code having P elementary codes with length N=2L. The new IGC codes have the properties described in Theorem 1, which are actually a special case of Z-complementary codes (Fan *et al.*, 2007). A lower bound on Z-complementary codes was established (Fan *et al.*, 2007) about the number of mates which is no larger than $P\lfloor N/Z\rfloor$, where P and N denote the number and the length of elementary codes respectively, and Z denotes the ZCZ length of the Z-complementary code set.

Let q and r denote two nonnegative integers such that $L=qZ+r$ where $0\le r<Z$. The length of generated IGC codes is $N=2L=2qZ+2r$. Then the theoretical maximum number is

$$M_{\mathrm{t}}=P\lfloor N/Z\rfloor=P\lfloor(2qZ+2r)/Z\rfloor=2qP+P\lfloor 2r/Z\rfloor. \tag{25}$$

The actual number of the generated IGC codes is denoted by M_{a}=2MG=2MP as the maximum number of mates of the original PC equals the number of elementary codes, i.e., G=P. According the construction process and the condition $2<Z<L$ of shift sequences, we have two cases to discuss.

Case 1: Z is even. Note that

$$M=\left\lfloor\frac{L-2}{Z}\right\rfloor=\left\lfloor\frac{qZ+r-2}{Z}\right\rfloor=\begin{cases}q, & 2\le r<Z,\\ q-1, & r=0,1,\end{cases}$$

$$M_{\mathrm{a}}=2MP=\begin{cases}2Pq, & 2\le r<Z,\\ 2P(q-1), & r=0,1,\end{cases}$$

$$M_{\mathrm{t}}-M_{\mathrm{a}}=\begin{cases}0, & 1<r<Z/2,\\ P, & Z/2\le r<Z,\\ 2P, & r=0,1.\end{cases}$$

Case 2: Z is odd. Note that

$$M=\left\lfloor\frac{L-1}{Z}\right\rfloor=\left\lfloor\frac{qZ+r-1}{Z}\right\rfloor=\begin{cases}q, & 1\le r<Z,\\ q-1, & r=0,\end{cases}$$

$$M_{\mathrm{a}}=2MP=\begin{cases}2Pq, & 1\le r<Z,\\ 2P(q-1), & r=0,\end{cases}$$

$$M_{\mathrm{t}}-M_{\mathrm{a}}=\begin{cases}0, & 0<r<Z/2,\\ P, & Z/2\le r<Z,\\ 2P, & r=0.\end{cases}$$

It is obvious that the number of the generated IGC codes can reach or almost reach the theoretical bound.

6 Example

In this section, an example of the newly generated IGC code set is described according to the following four steps.

Step 1: Set Z=3 and select an original PC code set $\boldsymbol{A}$ that has G=4 groups each with P=4 elementary codes and length L=4 as follows:

$$\boldsymbol{A}=\begin{bmatrix}\boldsymbol{A}^0\\ \boldsymbol{A}^1\\ \boldsymbol{A}^2\\ \boldsymbol{A}^3\end{bmatrix}=\begin{bmatrix}+++-;++-+;+++-;--+-\\ +-++;+---;+-++;-+++\\ +++-;++-+;---+;++-+\\ +-++;+---;-+--;+---\end{bmatrix}.$$

Step 2: According to Z=3 and L=4, we have $M=\lfloor (L-1)/Z \rfloor=1$, and the shift sequence set E is generated by Eq. (7):

$$E=\{\boldsymbol{e}_0=[0, 2]\}.$$

Step 3: $U^{g,1}$ is obtained from the elementary code $\boldsymbol{A}^g$ and the shift sequences E where $\boldsymbol{U}_i^g$ is generated based on $\boldsymbol{A}^g$ and $\boldsymbol{e}_i$ as follows:

$$U^{g,1}=\{\boldsymbol{U}_0^0,\ \boldsymbol{U}_0^1,\ \boldsymbol{U}_0^2,\ \boldsymbol{U}_0^3\},$$

$$\boldsymbol{U}_0^0=\{+++-++-+;+-++-+++;+++-++-+;-+--+---\},$$

$$\boldsymbol{U}_0^1=\{++-++++-;+----+--;++-++++-;-++++-++\},$$

$$\boldsymbol{U}_0^2=\{+++-++-+;+-++-+++;---+--+-;+-++-+++\},$$

$$\boldsymbol{U}_0^3=\{++-++++-;+----+--;--+----+;+----+--\}.$$

Step 4: $U^{g,2}$ is obtained from the elementary code $\boldsymbol{A}^g$ and the shift sequence set E where $\boldsymbol{U}_i^g$ is generated based on $\boldsymbol{A}^g$ and $\boldsymbol{e}_i$ as follows:

$$\boldsymbol{U}_1^0=\{+-+++---;+++---+-;+-+++---;---+++-+\},$$

$$\boldsymbol{U}_1^1=\{+---+-++;++-+---+;+---+-++;--+-+++-\},$$

$$\boldsymbol{U}_1^2=\{+-+++---;+++---+-;-+---+++;+++---+-\},$$

$$\boldsymbol{U}_1^3=\{+---+-++;++-+---+;-+++-+--;++-+---+\}.$$

Now we obtain an IGC code set U who has G=4 groups U^g (g=0, 1, 2, 3), $U^g=U^{g,1}\cup U^{g,2}$, with each group having $2M$=2 codes $\boldsymbol{U}_i^g$ (i=0, 1, 2, 3), and each code having P=4 elementary codes $\boldsymbol{U}_{i,j}^g$ (j=0, 1, 2, 3) with length $N=2L$=8.

Without loss of generality, we take $\boldsymbol{U}_0^0$ to calculate the autocorrelation values. We have

$$R_{\boldsymbol{U}_0^0,\boldsymbol{U}_0^0}=[32,\ 0,\ 0,\ 16,\ 0,\ 16,\ 0,\ 0].$$

Without loss of generality, we take $\boldsymbol{U}_0^0$ and $\boldsymbol{U}_1^0$ from the same group U^0 to calculate the cross-correlation values. We have

$$R_{\boldsymbol{U}_0^0,\boldsymbol{U}_1^0}=[0,\ 0,\ 0,\ 16,\ 0,\ -16,\ 0,\ 0].$$

Without loss of generality, we take $\boldsymbol{U}_0^0$ and $\boldsymbol{U}_1^2$ from different groups U^0 and U^2 to calculate the cross-correlation values. We have

$$R_{\boldsymbol{U}_0^0,\boldsymbol{U}_1^2}=[0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0].$$

As for the set size, $M_t=M_a$=8.

7 Conclusions

In this paper, a general construction of inter-group complementary (IGC) codes is proposed based on perfect complementary codes, interleaving operation, and the orthogonal matrix. The generated IGC codes have the following correlation properties: (1) the autocorrelation sidelobe values of the codes are zeros in the zero correlation zone (ZCZ); (2) the cross-correlation values between different codes in the same group are zeros in the ZCZ; (3) the cross-correlation values between codes of different groups are zeros during the whole period. Moreover, the ZCZ length of the newly generated IGC codes is flexible, which can extend their applications due to the tradeoff between the ZCZ length and the number of codes. The analysis of the size of newly generated IGC codes shows that it can reach or almost reach the theoretical bound, which means the number can be or almost be the maximum value.

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