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# U-shaped energy loss curves utilization for distributed generation optimization in distribution networks

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**Abstract:** We propose novel techniques to find the optimal location, size, and power factor of distributed generation (DG) to achieve the maximum loss reduction for distribution networks. Determining the optimal DG location and size is achieved simultaneously using the energy loss curves technique for a pre-selected power factor that gives the best DG operation. Based on the network's total load demand, four DG sizes are selected. They are used to form energy loss curves for each bus and then for determining the optimal DG options. The study shows that by defining the energy loss minimization as the objective function, the time-varying load demand significantly affects the sizing of DG resources in distribution networks, whereas consideration of power loss as the objective function leads to inconsistent interpretation of loss reduction and other calculations. The devised technique was tested on two test distribution systems of varying size and complexity and validated by comparison with the exhaustive iterative method (EIM) and recently published results. Results showed that the proposed technique can provide an optimal solution with less computation.

**Key words:** Distributed generation (DG), Distribution networks, Loss reduction, DG optimization, Energy loss curves
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## 1 Introduction

Distributed generation (DG), defined as a small generator connected to a distribution network, has been given much attention in recent years due to the considerable advantages it offers, such as active power loss reduction, voltage improvement, emission reduction, investment deferral, and increased reliability. The maximum benefits of DG units are noticeable when operated and placed at the optimal size and location. Therefore, many approaches such as analytical (Wang and Nehrir, 2004; Gozel and Hocaoglu, 2009), heuristic (Nara *et al.*, 2001; Al Rashidi and Al Hajri, 2011), and hybrid (Soroudi and Ehsan, 2011; Soroudi *et al.*, 2011) have been proposed to deal with various issues resulting from the impact of DG connection.

Distribution networks are well known for their lower $X/R$ ratio and voltage level, and hence higher amount of energy loss compared with transmission systems. Therefore, electrical energy loss reduction has been and will remain one of the most important concerns for distribution network operators. In general, DGs are connected near the customer in distribution systems to provide a portion of active and/or reactive power so that the line current is reduced to minimize system losses. Apart from optimization network reconfiguration and reactive power support through capacitor placement, DG optimization to minimize either power loss or energy loss has attracted the interest of the research community in recent years.

Chiradeja and Ramakumar (2004) and Ochoa *et al*. (2006) proposed a multi-objective criterion based on system performance indices, such as real and reactive power losses, system apparent power capacity enhancement, and voltage profile improvement for DG size and location planning with assumptions such as constant size or pre-selected locations. In the analysis, the importance of active and reactive power losses is observed. The sensitivity analysis relating the power loss due to DG current injection was used to allocate and size DG units (Kashem *et al.*, 2008). In Kashem *et al.* (2008), loads in the tested systems were assumed to be uniformly distributed and the location of the DG was based on the assumption of downstream load buses that are not common or appropriate for different distribution networks. Acharya *et al.* (2006) presented an analytical approach based on an exact loss formula to find the optimal size and location of the DG. In this method, the power flow calculation is employed only twice, and the approach is applicable to DGs capable of delivering only real power. This loss formula was further developed by Hung *et al.* (2010) to find the optimal sizes and optimal locations of four types of DG due to the active and reactive power injection. Also, it is shown that the optimal DG power factor is close to the power factor of the combined load of the respective system. Hung and Mithulananthan (2013) proposed an improved analytical (IA) method to achieve a high loss reduction in large-scale primary distribution networks by placement of multiple DG units. Biswas *et al.* (2012) presented a new formulation for the optimum DG placement problem, which determines the number, sizes, and locations of the DGs for a given low voltage distribution. The proposed optimization is based on a hybrid combination of technical and economic factors, e.g., minimization of networks power losses, reduction in voltage dip, and minimization of installation and maintenance costs of DGs.

Although minimization of power losses through DG employment has been extensively discussed in the literature, most of the studies cater only to a single load level. Hence, it is not possible to determine the actual impact of various types of DG (especially renewable types) or time variant loads on the system.

Porkar *et al.* (2011) proposed an attractive mathematical distribution system planning model considering integration of DG and load curtailment options in a competitive electricity market. They have shown that some effects of DG on the distribution system such as voltage profile improvement and loss reduction will play an increasing role in the electrical power system of the future. Ochoa *et al.* (2010; 2011) and Ochoa and Harrison (2011) have shown that the DG operation control has a great role in DG optimization planning, in areas such as energy losses, capacity assessment and reactive support for distribution networks. In their studies, both generation and demand were considered variable and only the size and reactive power production capacity of DGs varied in the pre-selected buses. Madureira and Lopes (2009) proposed a new method to minimize energy loss with support of DG units and micro-grids in distribution networks. Although the energy loss calculated is based on hourly network load variations, combined heat and power (CHP) units and wind turbines as DG units are fixed in the selected buses, so only their size and operation are optimized through minimizing the total energy losses of the network. Moradi and Abedini (2012) proposed a combined genetic algorithm and particle swarm optimization for optimal location and sizing of DG on distribution systems, to optimize radial distribution systems by minimizing network power losses, regulating bus voltages, and improving the voltage stability within the framework of system operation and security constraints in radial distribution systems.

Recently, an analytical approach based on sensitivity tests and the curve-fitting technique has been presented for finding the optimal size, location, and power factor of DGs (Abu-Mouti and El-Hawary, 2011). The DG deployment problem was divided into two stages. The sensitivity test with a candidate bus selection criterion is the first stage. It is shown that the load that causes the largest portion of the system losses is the most appropriate location for the DG. In the next stage an optimal DG size is determined by the curve-fitting technique on the selected bus. The best DG size is selected by comparing the results obtained in the second stage for four different power factors.

Note that analyzing bus sensitivities to power losses will not always lead to optimal DG location selection (Acharya *et al.*, 2006; Hung *et al.*, 2010; Hung and Mithulananthan, 2013). Therefore, using this method as the primary stage in DG optimization will also directly affect the second stage, i.e., DG sizing, and hence the optimization process can be totally changed. Though this problem has been somewhat alleviated by selecting buses with higher sensitivity to power losses (Anwar and Pota, 2011), the problems of changes in bus sensitivities and DG sizing for different load levels are still unsolved. Also, in this method, the optimization is implemented only for the DG with a unity power factor, despite the great influence of reactive power injection in the reduction of network losses.

To overcome these problems, we improve the heuristic optimization technique (Abu-Mouti and El-Hawary, 2011) for calculation of the optimal location and size of the DG by minimizing network energy losses simultaneously and use the method proposed by Hung *et al.* (2010) for optimal power factor selection. Based on the network's total load demand in percentage, four DG sizes with pre-selected optimal power factors are used to obtain the optimal size and location of DG. Also, the proposed algorithm is adopted to minimize energy losses by optimally accommodating and sizing of DG in the network with time varying load demand. Finally, this technique is applied to two test networks, and validated against recently published results obtained from various approaches.

## 2 Problem formulation

Distributed generation planning and operation consist of various linear and non-linear sub-problems. The area of concern is to propose a plan that satisfies the technical constraints at an optimum level and minimizes the total losses of the system associated with such a plan. The assumption, constraints, and evaluation objective function are described in the following.

### 2.1 Assumption

#### 2.1.1 Load modeling

The use of load patterns for billing and settlement activities in retail and wholesale markets is common (Chang and Lu, 2003; Chicco *et al.*, 2004). However, its application in the analysis of DG effects in distribution networks has received little attention.

In networks with time varying load demand, robust assessment of power flows is often based on hourly historical demand time series recorded for at least a year. Optimization and the power flow calculation for every hour of the day over a year result in a significant computational burden. In this study, to eliminate this burden, the daily load pattern is assumed to be the same throughout the year. Fig. 1 shows the average real and discrete modes of hourly demand snapshots for commercial customers in Iran from Feb. 20, 2011 to Mar. 20, 2011. The discrete original data is divided into a series of six levels covering specific hours.

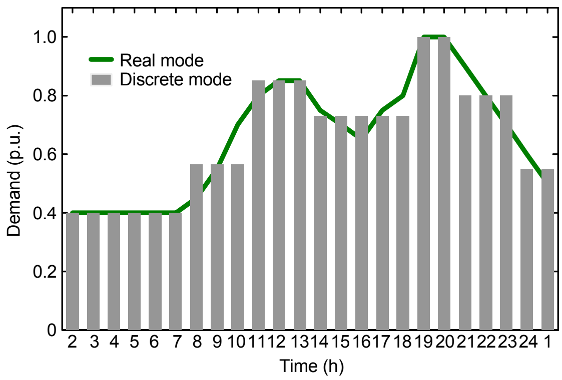


**Fig. 1 Average real and discrete modes of hourly demand snapshots for commercial customers in Iran during Feb. 20, 2011 to Mar. 20, 2011**

#### 2.1.2 Distributed generation modeling

In this study, a non-renewable generator (such as gas turbine or CHP) is considered as the DG source which is modeled in simulation as a negative constant load. Normally, this type of DG is able to operate at any desired output power and has the capability of controlling the reactive power (leading/lagging power factors). However, here it is assumed that the DG operates at a fixed maximum output power. The output active and reactive power of a controllable DG unit is modeled as

$$P_i^{\rm DG}=\cos\varphi^{\rm DG}\cdot S_{\max,i}^{\rm DG},\quad Q_i^{\rm DG}=\sin\varphi^{\rm DG}\cdot S_{\max,i}^{\rm DG},\quad(1)$$

where $P_i^{\rm DG}$, $S_{\max,i}^{\rm DG}$, and $Q_i^{\rm DG}$ denote the maximum apparent, active, and reactive power generated by the DG unit at bus *i*, respectively.

### 2.2 Constraints

#### 2.2.1 Power flow constraints

The most viable constraints are the well known power flow equations that need to be applied for each configuration and demand level. In this work, power flow analysis based on the Newton-Raphson equations (Tong and Miu, 2005) is realized to calculate the branch currents and the bus voltages with the existence of the DG as indicated in Eq. (2):

$$\begin{cases} P_{i,h}=P_i^{\mathrm{DG}}-P_{i,h}^{\mathrm{D}},\ Q_{i,h}=Q_i^{\mathrm{DG}}-Q_{i,h}^{\mathrm{D}}, \\ P_{i,h}=\left|V_{i,h}\right|\sum_{k=1}^{n}\left|V_{k,h}\right|\left[g_{ik}\cos(\theta_{i,h}-\theta_{k,h})+b_{ik}\sin(\theta_{i,h}-\theta_{k,h})\right], \\ Q_{i,h}=\left|V_{i,h}\right|\sum_{k=1}^{n}\left|V_{k,h}\right|\left[g_{ik}\sin(\theta_{i,h}-\theta_{k,h})-b_{ik}\cos(\theta_{i,h}-\theta_{k,h})\right], \end{cases} \tag{2}$$

where $V_{i,h}$ is the voltage magnitude of bus $i$ at demand level $h$, $\theta_{i,h}$ is the voltage angle of bus $i$ at demand level $h$, $P_i^{\mathrm{DG}}$ and $Q_i^{\mathrm{DG}}$ are the active and reactive power of a DG in bus $i$, respectively, $P_{i,h}^{\mathrm{D}}$ and $Q_{i,h}^{\mathrm{D}}$ are the active and reactive power of load in bus $i$ and at demand level $h$, respectively, $P_{i,h}$ and $Q_{i,h}$ are the net active and reactive power injected in bus $i$ and at demand level $h$, respectively, and $b_{ik}$ and $g_{ik}$ are the real and imaginary parts of admittance between buses $i$ and $k$, respectively.

2.2.2 Voltage limits

The magnitude of voltage at each bus $i$ with demand level $h$ should be kept within the safe operating limit described by

$$V_{\min}\le V_{i,h}\le V_{\max}, \tag{3}$$

where $V_{\min}$ and $V_{\max}$ are the lower and upper bounds of the bus voltage, respectively.

2.2.3 Maximum apparent power of feeders and the substation

The flow of current through the feeders and the substation should be kept below their thermal limit:

$$I_{\ell,h}\le I_{\max}^{\ell}, \tag{4}$$

where $I_{\ell,h}$ and $I_{\max}^{\ell}$ are the current magnitude of feeder $\ell$ at demand level $h$ and the thermal limit of feeder $\ell$, respectively.

2.2.4 Limitation on the power factor and size of DG

The apparent capacity and power factor of the DG should be kept within limitation as follows:

$$\mathrm{PF}_{\min}^{\mathrm{DG}}\le \mathrm{PF}^{\mathrm{DG}}\le \mathrm{PF}_{\max}^{\mathrm{DG}}, \tag{5}$$

$$S^{\mathrm{DG}}\le\sum_{i=1}^{n}S_{i,h}^{\mathrm{D}}. \tag{6}$$

In this study, the lower and upper power factor limits are 0.85 (leading) and unity, respectively. Furthermore, considering that reverse power flow can produce a significant modification of the voltage profile, the maximum capacity of the DG should be lower than the total load of the network at any demand level to ensure the normal direction of power flow.

### 2.3 Active losses and the objective function

The total system active power loss at demand level $h$ is represented by an exact loss formula:

$$P_{\mathrm{Loss},h}=\sum_{i=1}^{n}(P_{i,h}+P_h^{\mathrm{grid}}), \tag{7}$$

where $P_h^{\mathrm{grid}}$ is the active power imported from the grid at demand level $h$.

By considering time variant loads in the network, total system power losses would be changed to total system energy losses at all demand levels. Therefore, minimization of the total energy losses as an objective function while satisfying all constraints in Eqs. (2)–(6) can be expressed as

$$\min E_{\mathrm{Loss}}=365\sum_{h=1}^{n_{\mathrm{d}}}P_{\mathrm{Loss},h}\tau_h, \tag{8}$$

where $\tau_h$ is the duration of demand level $h$ and $n_{\mathrm{d}}$ is the number of demand levels in a day.

## 3 Proposed optimization technique

The search for the exact global solution to the problem of locating and sizing DGs requires immense computational time. Most reported works to date are based on assumptions such as choosing a fixed DG size, selecting a DG with a unity power factor, or selecting some buses for possible DG connection. Although these assumptions reduce the search space and achieve reasonable results for particular applications, they do not provide concrete results. Hence, the capability of any algorithm in handling various DGs in terms of size, location, operation, etc. is vital.

In this work, a heuristic optimization methodology based on the curve-fitting technique described in Abu-Mouti and El-Hawary (2011) is developed for simultaneous determination of the DG location and size, with time-varying loads. Abu-Mouti and El-Hawary (2011) showed that the total system real power loss due to different DG sizes can be represented by a quadratic function. Since the total system real power loss is dependent on load demand, the related quadratic function would change for various demand levels. By supposing installation of a DG with a firm output in one of the buses of a simple test system (Ghosh *et al.*, 2010) with variable demand (three levels), the quadratic function related to the total system real power loss varies (Fig. 2).

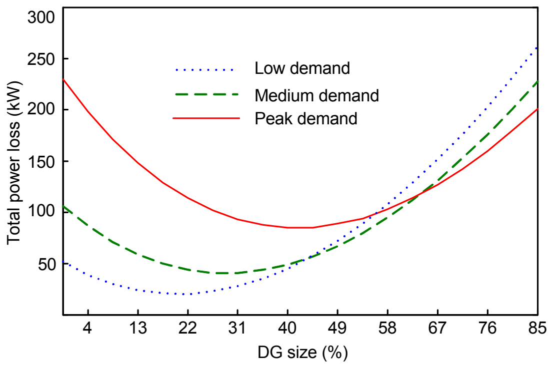


**Fig. 2 Total system real power loss as a function of DG size (represented as the ratio of DG power injection to the total load) at three demand levels**

According to Fig. 2, although the total power loss is changing at different demand levels, a distinct U-shape is formed for each level. The U-shape is evident as the power injected by the DG capacity initially reduces the system's power loss. However, when it exceeds the optimal rate, the line current starts to increase due to the large power injected by the DG, which increases the system loss. Also, Fig. 2 shows that the loss benefit from choosing the optimal rate of the DG varies between different demand levels. The peak, medium, and low demand analysis results show that the network loss is directly proportional to the demand level as well as the optimal size of DG. Therefore, in networks with time variant loads, the DG optimization planning based on reduction in the network's power loss may overestimate or underestimate the size of the DG. As a result, DG sizing would be more realistic when the objective function changes from the minimization of power losses to energy losses. In energy analysis, by assuming that the duration of each load level is equal (2920 h per year), the annual energy loss relative to the delivered power from the DG is as shown in Fig. 3.

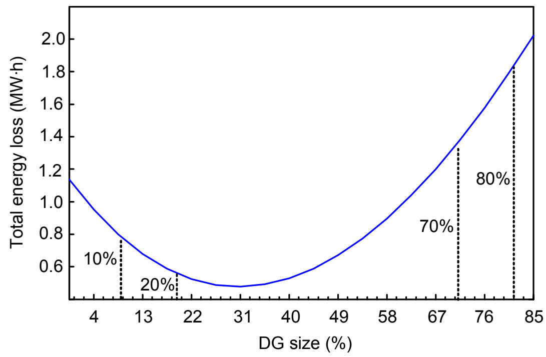


**Fig. 3 Total system energy loss as a function of DG size (represented as the ratio of DG power injection to the total load)**

Fig. 3 shows that the variations of energy loss relative to DG power injection in bus *i*, which is a quadratic function ($f_i$=$E_{\text{loss}}(S_{\text{DG}})$), are similar to the power loss curves shown in Fig. 2. Recognizing this function for all buses will yield some information about the network that is useful in the DG optimization problem. For example, finding the minimum point on the energy loss curve of each bus will reveal the optimum size of the DG. Also, the sensitivity of the network's energy loss to power injection for each bus is determined by the gradient of the above mentioned function when DG size varies between the initial point (10% of total load) and the lowest point of the curve.

Considering the importance of this function, the following conditions are essential in implementing our proposed approach:

1. Plot $f_i$: The optimal region of DG size falls mostly within 20%–70% of the total load of the network. Therefore, to plot a quadratic function curve

with high precision, as in Fig. 3, additional points on either side of 20% and 70% are required, which are 10% and 80% of the total load. These values are carefully selected based on experimental results (Abu-Mouti and El-Hawary, 2011). Then, the lowest point on the curve is found by equating $\frac{\partial f_i}{\partial S_{\mathrm{DG}}}$ to zero, which leads to the optimum size of the DG on bus *i*.

2. DG power factor: DG operation (i.e., real and reactive power dispatches) is an important issue for network loss minimization. In this work, determination of the optimum DG power factor is based on the concept known as the 'fast approach', which was discussed in Hung and Mithulananthan (2013). Based on this approach, the initial power factor of the DG is selected to be equal to the power factor of the combined load of the network. Minimum loss occurs when the DG injects both active and reactive power and operates in the leading mode.

The computational procedure to optimize DG size and location based on the U-shaped energy loss curve and the DG power factor is described in detail as follows:

Step 1: Read load data, system topology, and line data.

Step 2: Select the four DG sizes (10%, 20%, 70%, and 80% of their nominal capacities) based on the total load of the network.

Step 3: Identify the DG power factor based on the 'fast approach'.

Step 4: Run power flow for all buses with each value of DG in step 2 with a selected power factor in step 3, and register the total energy losses for each combination.

Step 5: Form the curve function expressed by Eq. (9) for each bus from the registered results:

$$f_i(x) = ax_i^2 + bx_i + c, \tag{9}$$

where $f_i$ and $x_i$ are the total energy loss and the DG size at bus *i*, respectively.

Step 6: Find the optimum point of the curve ($x_i^{\mathrm{opt}}$) for each bus using

$$\frac{\partial f_i}{\partial x} = 0 \rightarrow x_i^{\mathrm{opt}} = -\frac{b}{2a}, \tag{10}$$

where $x_i^{\mathrm{opt}}$ is the optimum size of the DG at bus *i*, with which the total energy loss will be the minimum. For example, $x_2^{\mathrm{opt}}$=1200 kV·A represents that the optimum size of the DG at bus 2 is 1200 kV·A.

Step 7: Substitute $x_i^{\mathrm{opt}}$ for each function in Eq. (9) or run the power flow with $x_i^{\mathrm{opt}}$ and then find the minimum energy loss at each bus ($f_i^{\min}$). Therefore, two parameters $x_i^{\mathrm{opt}}$ and $f_i^{\min}$, which represent the optimum size of the DG and the minimum energy loss at bus *i*, respectively, are assigned to bus *i*.

Step 8: Rank buses in ascending order of values obtained in step 7 to form the priority bus. Then, select the highest priority which is the minimum energy loss.

Step 9: Place a DG with $x_i^{\mathrm{opt}}$ capacity at the selected bus in step 8.

Step 10: Run power flow with selected DG size and location. In the obtained results, if the constraints are within limits (i.e., the optimum DG size and location), go to step 11; otherwise, one of the following three scenarios should be performed:

1. If the voltage at a particular bus at any demand level is beyond the upper limit or the flow of current through each feeder exceeds its thermal limit, reduce the DG size in 'small' steps and repeat step 10 with this new capacity.

2. If the obtained DG size is more than the total network load at any demand level, reduce the DG size in 'small' steps and repeat step 10 with this new capacity.

3. If the voltage at a particular bus at any demand level is lower than the minimum limit, increase the DG size in 'small' steps and repeat step 10 with this new capacity.

Step 11: If the energy loss of the system is still the highest in the priority list (the minimum energy loss), the DG size, location, and power factor are determined optimally in the network; otherwise, remove this bus from the priority list and repeat the whole process from step 8.

Note that this technique can be used at any power factor by placing the desired power factor in step 3.

Fig. 4 shows the flowchart of the proposed optimization method.

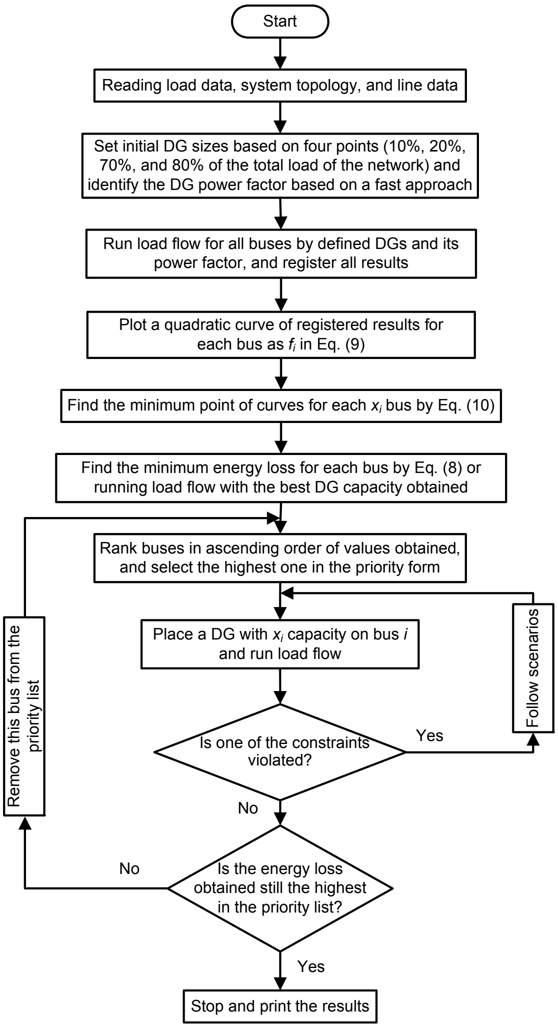


**Fig. 4 Flowchart of the proposed technique for distributed generation (DG) optimization**

## 4 Numerical results and discussion

### 4.1 Test systems

Two test systems with different sizes and complexities have been used to explain, check, and validate the proposed technique for identification of the optimal size, location, and power factor. The first system is a 17-bus radial test distribution system with a total real and reactive demand power of 13.88 MW and 5.64 Mvar, respectively (Mendoza *et al.*, 2007). The second one is a 33-bus radial test distribution system with a total real and reactive demand power of 3.71 MW and 2.3 Mvar, respectively (Abu-Mouti and El-Hawary, 2011).

First, the operation mechanism of the proposed algorithm is described thoroughly given the simplicity of the 17-bus system.

Then, due to the wide use of the second system in the literature, the results attained using the proposed technique for this system will be compared with those obtained using other methods. In addition, results for both systems will be compared to those obtained using the exhaustive iterative method (EIM).

Based on the proposed method, an analytical software tool has been developed in a MATLAB environment for power flow, determining the optimal location, size, and power factor of DG in distribution networks.

### 4.2 Scenario development

Two comprehensive scenarios are analyzed to cover the different demand characteristics in the DG optimization problem:

Scenario A: maximum demand. It is assumed that the demand and DG output are fixed at the maximum level during the study period. With these assumptions, DG optimization can be achieved by considering the power loss, instead of energy loss, as an objective function.

Scenario B: variable demand. It is assumed that the customers of the network are commercial and their pattern of demand is variable at six levels as shown in Fig. 1. Hence, energy analysis as opposed to power analysis is necessary when there is a variation of demand during the study period.

### 4.3 Seventeen-bus feeder system

Fig. 5 shows the 17-bus feeder system. The total load of this system is 14.98 MV·A. According to the proposed method, the four pre-selected DG sizes are 1.5, 3, 10.5, and 12 MV·A. Also, based on the ‘fast approach’, the power factor of the DG is assumed as the power factor of combined load demand, 0.92 in the leading mode.

In the following, two scenarios A and B are studied using the proposed method and EIM to optimize DG location, size, and power factor.

#### 4.3.1 Scenario A

For scenario A, by considering the load demand as a constant load at the peak level, and placing the DG with the four mentioned sizes and an assumed power factor in all the buses of the network, the total energy loss of the network would follow the U-shaped curves shown in Fig. 6.

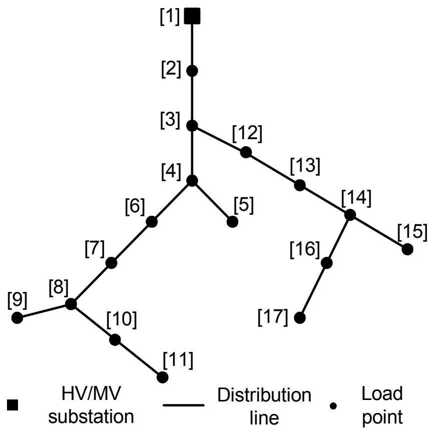


**Fig. 5 Single line diagram of the 17-bus feeder**

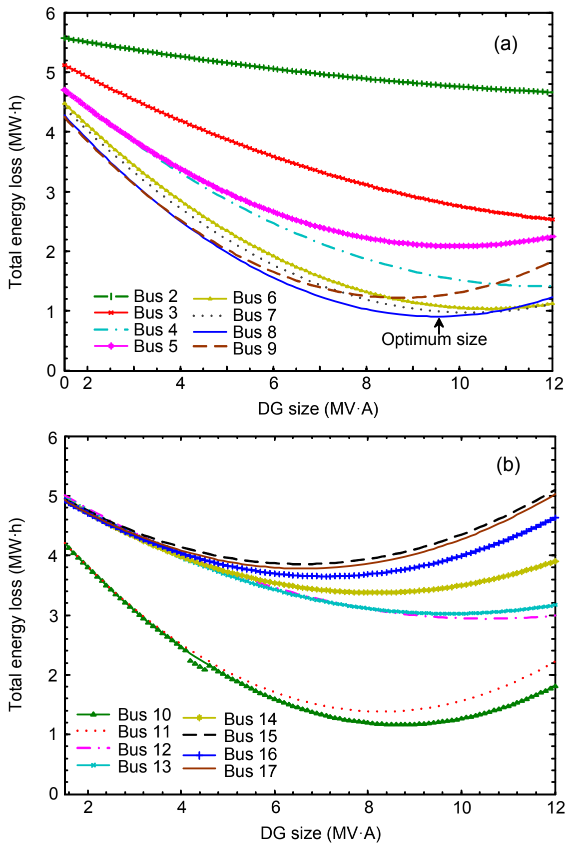


**Fig. 6 Variation of total energy loss curves in buses 2–9 (a) and buses 10–17 (b)**

Power injection into the network from each bus has its own unique behavior. Also, variation of network energy loss depends on the specific characteristics of the network, such as demand distribution, topology, as well as the location of generators. According to the results shown in Fig. 6, the energy losses of the system are reduced more when DG is installed on buses 6 to 11. This is related to the demand of this part being close to half of the total demand of the network. Although DG installation at the last nodes of the network is supposed to be the most appropriate location, Fig. 6 suggests that due to low demand on buses 14 to 17, placement of the DG in these buses has no effect on loss reduction. This is also true for buses 4 and 5 in the center of the network. In these buses, if the DG size increases beyond 7 MV·A, a large portion of power injected by the DG would be reversed, increasing the network loss.

Furthermore, the first buses in the network such as buses 2 and 3 are not appropriate for DG installation. This is due to these buses being close to the slack bus, and power injection in these buses will not have any effect on loss reduction.

Based on the proposed technique and the results shown in Fig. 6, the best location and size of the DG would be bus 8 with a capacity of 9524 kV·A, which leads to minimum total network energy loss per year with no constraint violated.

### 4.3.2 Scenario B

In scenario B, by assuming that all customers are commercial, the demand is considered as variable at six levels (Fig. 1). Based on the proposed method, the variation of network energy loss against DG size is calculated for all busses. Fig. 7 shows the results for selected buses (which have higher priorities).

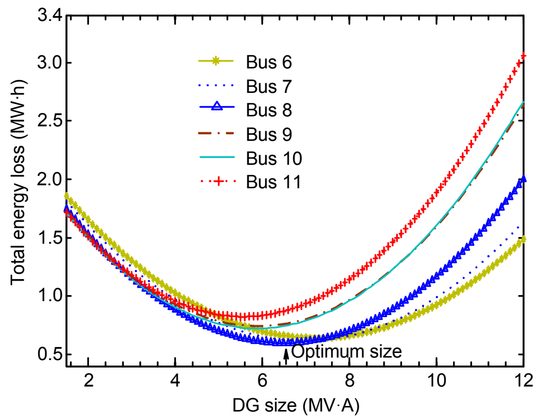


**Fig. 7 Variation of total energy loss in the selected buses**

Comparison of Fig. 7 with Fig. 6 suggests that the optimum size of DG for each bus decreases by more than 2 MV·A. In the case of scenario B, although the best DG location is also bus 8, the optimum size decreases to 6486 kV·A.

Table 1 shows a comparison of the simulation results (optimum DG size and location) for the two scenarios using the proposed approach and EIM.

According to the results shown in Table 1, for scenario A, the optimal location, size, and power factor of the DG unit obtained using the proposed method are very similar to those obtained using EIM.

In scenario B, the optimum DG size obtained using the proposed method is 6036 kV·A as opposed to 6082 kV·A calculated using the EIM method (Table 1, Fig. 7). However, the DG location, power factor, and percentage of loss reduction are the same.

Note that the initial optimum DG size obtained from the U-shaped curves (proposed method) is 6486 kV·A. However, installation of this size of DG at bus 8 leads the current through the substation to be reversed during the lowest demand (the constraint in Eq. (6) is violated). Therefore, to satisfy all the constraints, the DG size decreases to 603 kV·A in small steps. The minimum and maximum voltages in scenario B are related to the heaviest and lightest demand levels, respectively. Study of these two scenarios shows that by considering variable demand, the optimum DG size decreases, as well as the total energy loss.

### 4.4 Thirty-three-bus feeder system

Fig. 8 shows the second test feeder which is a 12.66 kV, 33-bus, 32-branch, radial distribution system with a total demand of 4.37 MV·A.

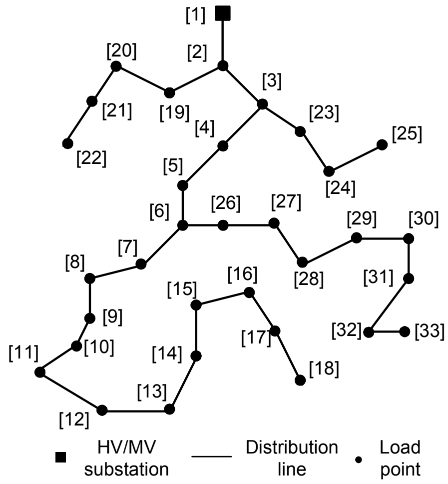


**Fig. 8 Single line diagram of the 33-bus feeder**

According to the proposed method and network's total demand, the four pre-selected DG sizes are 430, 900, 3060, and 3500 kV·A. Also, based on all active and reactive system demands, the power factor of the DG is assumed to be 0.85 in leading mode. Like in Section 4.3, two scenarios A and B are studied using the proposed method and EIM to optimize the DG location, size, and power factor.

#### 4.4.1 Scenario A

Table 2 shows the simulation results of the optimal size, location, and power factor of the DG unit using the proposed method in scenario A. The results of this method and EIM at the optimal power factor are compared. Based on the 'fast approach', the optimal power factor of the DG unit is determined to be equal to the combined load power factor at 0.85 and in leading mode.

**Table 1 Distributed generation (DG) placement by various techniques for the 17-bus network in scenarios A and B**

| Scenario | Method | Location (bus No.) | Power factor (leading) | DG size (kV·A) | DG size[#] (%) | Total energy loss (MW·h) | Loss reduction (%) | $\lvert V_{\min}\rvert^{*}$ (p.u.) | $\lvert V_{\max}\rvert^{**}$ (p.u.) |
|---|---|---|---|---|---|---|---|---|---|
| A | Proposed | 8 | 0.92 | 9524 | 63.5 | 940.6 | 83.7 | 0.9799 | 1.0005 |
| | EIM | 8 | 0.92 | 9547 | 63.7 | 940.6 | 83.7 | 0.9800 | 1.0006 |
| B | Proposed | 8 | 0.92 | 6036 | 43.3 | 599.4 | 77.3 | 0.9723 | 1.0156 |
| | EIM | 8 | 0.92 | 6082 | 40.6 | 598.3 | 77.3 | 0.9724 | 1.0160 |

[#] Represented as the ratio of DG power injection to the total load. [*] At bus 17; [**] at bus 8

**Table 2 Distributed generation (DG) placement by various techniques for the 33-bus network in scenarios A and B**

| Scenario | Method | Location (bus No.) | Power factor (leading) | DG size (kV·A) | DG size[#] (%) | Loss reduction (%) | $\lvert V_{\min}\rvert^{*}$ (p.u.) | $\lvert V_{\max}\rvert$ (p.u.) |
|---|---|---|---|---|---|---|---|---|
| A | Proposed | 6 | 0.85 | 3051 | 69.7 | 69.6 | 0.9662 | 1.0008[**] |
| | EIM | 6 | 0.82 | 3088 | 70.7 | 69.7 | 0.9668 | 1.0013[**] |
| B | Proposed | 26 | 0.85 | 1765 | 40.4 | 61.8 | 0.9446 | 1.0011[##] |
| | EIM | 26 | 0.82 | 1832 | 41.9 | 62.3 | 0.9457 | 1.0120[##] |

[#] Represented as the ratio of DG power injection to the total load. [*] At bus 18; [**] at bus 6; [##] at bus 26

Results obtained from the proposed method and the EIM approach correlate well in terms of the optimal DG size, location, and power factor. The insignificant difference in loss reduction values between the two methods supports the claim that the proposed technique offers enhanced results.

Fig. 9 shows the difference between bus voltages of the 33-bus system before (default) and after using the retained DG options obtained using the proposed method and EIM.

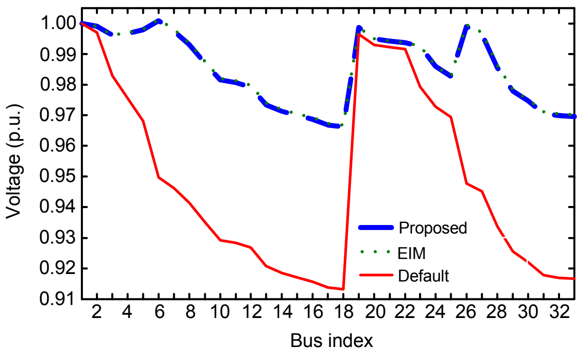


**Fig. 9 Voltage profiles without DG and with DG placement by the proposed technique and EIM**

Fig. 9 shows that the small differences in the obtained power factors and DG power outputs between the proposed method and EIM have no significant effect on network bus voltages.

Also, a close inspection of the results shown in Fig. 9 reveals that despite meeting the voltage constraint in bus 6 in the initial network configuration (without DG) and being close to the slack bus, this bus is the most suitable location to install the envisaged DG unit which, considering the objective function, leads to significant reduction in power network losses. Also, note that due to the relatively great distance between bus 6 and the buses near the end of the network, installing the DG unit at bus 6 will greatly improve the voltage profile of those buses, and all voltages in the network will come within required limits.

### 4.4.2 Scenario B

For scenario B, it is assumed that the customer demand varies at six levels (Fig. 1). Therefore, all demand levels should be considered in the DG optimization problem. Table 2 shows the simulation results of the optimal size, location, and power factor of the DG unit using the proposed method and EIM in scenario B.

According to the results obtained, in this scenario, although the DG size and power factor obtained using the proposed method are different from those obtained using EIM, the compensated results of methods are very close. In other words, the additional enhanced results achieved using the EIM solution are better by only 0.5% in loss reduction and 0.0011 p.u. in system's $|V_{\min}|$, which supports the achievement of the proposed technique.

Comparison of the results obtained in scenarios A and B (Table 2) shows that considering the demand pattern as a model for a practical situation, energy analysis is very effective in DG optimization. For example, in addition to changing the optimal DG location from buses 6 to 26, the best DG size obtained in the peak demand (scenario A) would be higher than that in the real situation (scenario B) by 1281 kV·A. Therefore, in scenario A, loss reduction and $|V_{\min}|$ would be more than those in reality by 7.8% and 0.0216 p.u., respectively.

### 4.4.3 Comparative study

The best DG size and location of the proposed method for the 33-bus feeder system are compared with those obtained using the solutions proposed by Hung *et al.* (2010), Abu-Mouti and El-Hawary (2011), and Hung and Mithulananthan (2013). Based on the 'fast approach' in the proposed method, the optimal power factor of the DG unit is determined to be equal to the combined load power factor at 0.85 and in leading mode. Also, as the load is considered constant in Hung *et al.* (2010), Abu-Mouti and El-Hawary (2011), and Hung and Mithulananthan (2013), the results from the first scenario are compared to those from the literature (Table 3). Results obtained using the proposed method, Hung *et al.* (2010)'s method, and Hung and Mithulananthan (2013)'s method correlate well in terms of the optimal DG size, location, and power factor. In addition, Table 3 shows that finding DG options optimally using the proposed method, Hung *et al.* (2010)'s method, and Hung and Mithulananthan (2013)'s method, leads to significant loss reduction (around 70%) while satisfying all the power and voltage constraints.

Another result for comparison is the optimum DG options suggested by Abu-Mouti and El-Hawary

**Table 3 Distributed generation (DG) placement by various techniques for the 33-bus network**

| Method | Location (bus No.) | Power factor (leading) | DG size (kV·A) | DG size# (%) | Loss reduction (%) | $\lvert V_{min}\rvert^*$ (p.u) | $\lvert V_{max}\rvert$ (p.u) |
|---|---|---|---|---|---|---|---|
| Hung *et al.*, 2010 | 6 | 0.85 | 3025 | 69.2 | 69.6 | 0.9658 | 1.0004** |
| Hung and Mithulananthan, 2013 | 6 | 0.82 | 3107 | 71.1 | 69.7 | 0.9671 | 1.0016** |
| Proposed method | 6 | 0.85 | 3051 | 69.7 | 69.6 | 0.9662 | 1.0008** |
| Abu-Mouti and El-Hawary, 2011 | 29 | 0.85 | 2000 | 45.8 | 67.3 | 0.9488 | 0.9989## |

# Represented as the ratio of DG power injection to the total load. * At bus 18; ** at bus 6; ## at bus 29

(2011), which are based on the heuristic curve fitting technique. Although their technique is similar to the basis of the proposed method, the results obtained are different. In fact, using sensitivity analysis in Abu-Mouti and El-Hawary (2011) is not appropriate for finding the optimum DG location and hence effects on DG size optimization. Therefore, the method proposed by Abu-Mouti and El-Hawary (2011) yields the lowest loss reduction due to poor choice of DG location and size, as shown in Table 3.

## 5 Conclusions

This paper has presented a heuristic simultaneous method for finding the optimal location, size, and power factor of the DG for energy loss reduction in distribution networks. The method is based on the total energy loss curves that reduce the search space significantly through requiring only four DG tests for each bus of the network. Furthermore, the optimal power factor of the DG is assumed to be equal to the combined load power factor of the system. Hence, this method could be easily applied in DG optimization planning in large scale distribution networks. The validity of the proposed analytical technique for finding optimal DG solutions is explained and verified on two distribution networks with varying sizes, locations, and power factors using an exhaustive iteration method and recently published solutions.

Results show that all the DG factors, namely location, size, and power factor, are crucial factors in minimizing power or energy loss of the system. If these factors are selected appropriately, the distributed generation can reduce the total losses significantly. In contrast, an improper DG selection can lead to increasing network losses, even above pre-DG installation. Also, DGs that are capable of delivering both active and reactive power reduce losses more than those delivering only real power. In the 17- and 33-bus feeders, the operating power factor of the DG unit for minimizing energy losses is found to be very close to the power factor of the combined load of the system. Furthermore, in this study power loss reduction is analyzed alongside the energy losses in all the test systems. The main analysis shows that the pattern of demand is very effective in DG optimization. According to the results obtained, the practice of minimizing power losses by examining only a single (peak) load condition is unlikely to lead to an overall optimal energy reduction.

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