Journal of Zhejiang University-SCIENCE C (Computers & Electronics)
ISSN 1869-1951 (Print); ISSN 1869-196X (Online)
www.zju.edu.cn/jzus; www.springerlink.com
E-mail: jzus@zju.edu.cn

Feasibility analysis for attitude estimation based on pulsar polarization measurement*

Nan LUO†, Lu-ping XU, Hua ZHANG, Qiang XIE
(*School of Electronic Engineering, Xidian University, Xi'an 710071, China*)
†E-mail: nluo@mail.xidian.edu.cn
Received Oct. 15, 2012; Revision accepted May 4, 2013; Crosschecked May 13, 2013

Abstract: One of the important characteristics of pulsar radiation is polarization. It is considered not only as a probe for recognizing the structure of a magnetic field, but also as a lighthouse for estimating spacecraft attitude via orientation information between the pulsar and the detector. Although polarization of a pulsar has been studied for decades, until recently applications to determination of spacecraft attitude have been seldom reported. This paper deals with analysis of the feasibility of applying polarization information to attitude estimation. The stability factor (SFR) and observation fluctuation factor (OFR) are introduced to analyze the stability of a pulsar's polarized position angle. Based on European Pulsar Network (EPN) data, several simulated instances are used to demonstrate that the accuracy requirement of attitude determination can be met via polarization measurement. The SFR of a pulsar is evaluated using simulated polarization data, and the OFR is used to analyze the relationship between fluctuation extent and observation time. Simulation results show that the polarized measurement of candidate pulsars PSR B0470-28 and PSR B2319+60 reaches the specification for attitude determination.

Key words: Pulsar, Polarization, Attitude determination, Stable factor
doi:10.1631/jzus.C1200291 **Document code:** A **CLC number:** TP2; V52

1 Introduction

Soon after discovering the first pulsar, it was found that radio emission from a pulsar is highly polarized. It is generally believed that polarization of pulsar radiation is determined by the geometry of open magnetic field lines in the magnetic-pole model, and the angle of the line of sight with respect to the spin axis of a pulsar can be deduced from the observed variations of the polarization position angle (PPA) with this model. Related to polarization is directionality of the pulsar radiation yield, which is an extraordinarily beneficial feature for application to attitude determination. Actually, this feature can be observed at optical (Jones *et al*., 1981; Smith *et al*., 1988; Slowikowska *et al*., 2009), radio (McKinnon and Stinebring, 1998; 2000; Petrova, 2001; Han *et al*., 2009), and X-ray (Silver *et al*., 1978; Weisskopf *et al*., 1978; Hughes *et al*., 1984; Camilo *et al*., 2007; Dean *et al*., 2008; Forot *et al*., 2008) frequencies, and in general, linear polarization is dominant across the whole profile (Gould and Lyne, 1998; Karastergiou and Johnston, 2007). Near polarimetric measurements of pulsars are performed in the radio domain. Fast X- and γ-ray polarimetry from space-borne instruments is presently of very limited sensitivity. Results have therefore been reported only for the brightest pulsar and its pulsar wind nebulae, e.g., the Crab. The Crab nebula is the only celestial source whose polarization characteristics in terms of the rotational phase are known in great detail and are up to date (Weisskopf *et al*., 1978; Dean *et al*., 2008). The highly linearly polarized emission is an important qualification by which the orientation of the spacecraft can be estimated without additional information. With the

* Project supported by the National Natural Science Foundation of China (No. 61172318) and the Fundamental Research Funds for the Central Universities, China (Nos. K5051302015 and K5051302040)

increasing development of cosmic ray detection technology, many missions have been proposed to enhance polarization exploration by either radio or X-ray wavelength, which will certainly drive the application of pulsar polarization features greatly. Two representative examples are the balloon-borne hard X-ray polarimetry with PoGOLite (Mizuno *et al.*, 2007; Pearce *et al.*, 2012) and the hard X-ray polarimeter X-Calibur (Krawczynski *et al.*, 2011).

An autonomous spacecraft navigation method was demonstrated by Bernhardt *et al.* (2011) using the timing properties of a pulsar. However, little research has been conducted with respect to applying the knowledge of polarization to attitude determination. It is reasonable to suppose that polarization can be used to estimate spacecraft attitude. Obviously, a close relationship can be established between directionality of polarization and detector attitude. To this aim, the profile of PPA needs to be exactly detected and analyzed statistically beforehand. In this paper, the model of PPA is first discussed, and then the statistical analysis of PPA and feasibility of attitude estimation using the polarization measurement is carried out. The effectiveness of the proposed method is demonstrated by numerical simulation.

2 Modeling for the polarization position angle

Since pulsar radiation is very weak and the individual pulses are desultory, an efficient way to obtain the standard pulse profile is to accumulate the observed signals cyclically. Similarly, the shape of the polarization angle is random for an individual pulse, but on a statistical basis, accumulation has proved a useful method for studying the variation regularity of PPA. By performing this method for polarization analysis, it has been found that the shape of PPA is periodically aptotic and corresponds to the accumulated profile (Morris *et al.*, 1981; Lyne and Manchester, 1988; Manchester and Johnston, 1995). The average profile of PPA accompanies with the smooth variation of the linear polarization angle through the pulse. Additionally, at a given observed frequency the linearly polarized position angle is not a function of frequency. A characteristic swing of the position angle (PA), i.e., an S-like shape near the pulse center, is interpreted in the rotating vector model (Radhakrishnan and Cooke, 1969) as a projection of the magnetic field line at the point of emission onto a plane perpendicular to the line of sight of the observer. This model was later corrected due to the relativistic effects (Blaskiewicz *et al.*, 1991). Fig. 1 shows the magnetic-pole model. The observed PPA ψ is given by (Lyne and Manchester, 1988; Johnston *et al.*, 2005)

$$\tan(\psi-\psi_0)=\frac{\sin\alpha\sin\phi}{\sin\zeta\cos\alpha-\cos\zeta\sin\alpha\cos\phi}, \quad (1)$$

where ψ_0 is the position angle of the projected direction of the pulsar rotation axis, α and ζ are the inclination angles of the magnetic axis and observer direction to the rotation axis respectively, and ϕ is the longitude, ϕ_0=0. For a large number of highly polarized pulsars, the observed position angle variation through the pulse can be effectively described by this model (Manchester and Taylor, 1977; Johnston *et al.*, 2005). The integrated profile and the mean polarization profile of the observed pulsars both remain steady from one pulse sequence to the next one. The linear polarization properties have no distinct change except that sometimes the polarization mode may change transiently.

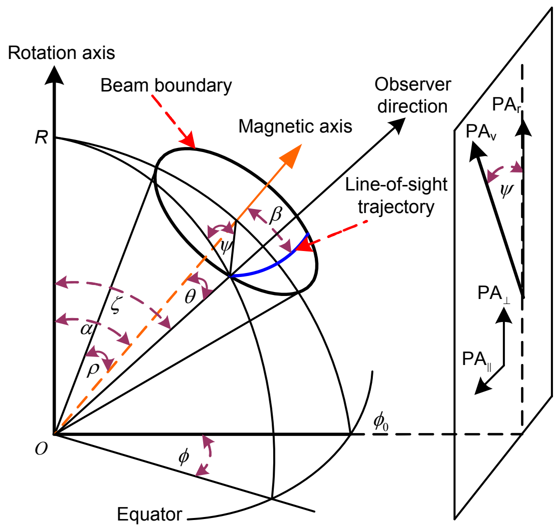

Fig. 1 Geometry of the magnetic-pole model for pulsars
β is the inclination of the line-of-sight to the magnetic axis, and its trajectory presents a cross through the beam boundary when the pulsar rotates. ρ is the opening semi-angle of the conical emission beam

The biomimetic method by which insects (e.g., bee) make use of the polarization of sky light to locate position and determine direction (Horváth and Varju, 1963; Labhart, 1988; Chu *et al*., 2008; Zhao *et al*., 2009) has been investigated for spacecraft navigation during the past years. Since few attempts have been made on using polarization features of pulsars to estimate attitude, we aim to analyze the feasibility and principle of attitude determination via polarization information. Considering a polarimeter as the biomimetic sensor, detector attitude can be determined by detecting the PPA of the pulsar owing to the directionality of polarization. Then combining this with the fixed coordinate relationship between the detector and the spacecraft, spacecraft attitude can be calculated. In other words, there exists an established relevance between spacecraft attitude and PPA. The Stokes parameters are employed to present the observation results of pulsar polarization, defined as (McMaster, 1954)

$$\boldsymbol{S}=(I,\ Q,\ U,\ V)^{\mathrm{T}}. \tag{2}$$

Here I is the total intensity of pulse, Q and U are orthogonal linearly polarized components of pulse, and V is the circular component of polarization. The formula of each component of polarization is given by means of the Stokes parameters, which imply the correlation between the observed data and PPA. The relationship between PPA and the Stokes parameters is given as

$$\begin{cases} Q=L\cos(2\psi), \\ U=L\sin(2\psi), \\ V=\sin\theta, \end{cases} \tag{3}$$

where L is the linear polarization component of pulse, ψ is the observed PPA, and θ is the inclination of the magnetic axis to the observer direction. Next, it is absolutely necessary to investigate the stability of the position angle in a distinct phase for applying polarization information to attitude determination.

3 Stability analysis and attitude estimation

A quality index η_{SF}, called the stability factor (SFR), is defined to evaluate the fluctuation in the position angle at a given longitude:

$$\eta_{\mathrm{SF}}=\frac{\left|P_{\mathrm{std}}(\phi)-P_{\mathrm{mean}}(\phi)\right|}{P_{\mathrm{std}}(\phi)}. \tag{4}$$

Here $P_{\mathrm{mean}}(\phi)$ is the mean intensity of the linearly polarized component of observed profiles in several time cycles at longitude ϕ. The standard profile $P_{\mathrm{std}}(\phi)$ is the predictive standard value of the linearly polarized component of the integrated profile at the same longitude, which is known beforehand. Theoretically, η_{SF} ranges from 0 to 1. An η_{SF} approaching 0 means a high stability of PPA, and an η_{SF} close to 1 means a large fluctuation.

An observation fluctuation factor (OFR) η_{OF}, which indicates the dispersion of $P_i(\phi)$ against $P_{\mathrm{mean}}(\phi)$, is also introduced to evaluate the fluctuation of the PPA profile:

$$\eta_{\mathrm{OF}}=\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left[P_i(\phi)-P_{\mathrm{mean}}(\phi)\right]^2}. \tag{5}$$

Here $P_i(\phi)$ represents the accumulated value in interval Δt_i at longitude ϕ, and $\Delta t_i=t_{i+1}-t_i$. $P_{\mathrm{mean}}(\phi)$ is the mean intensity of the linearly polarized component of observed profiles from t_1 to t_n.

SFR determines whether a mean profile accumulated during an observation is effective for estimating attitude. A small SFR indicates that the observed profile is close to the predicted standard profile in the catalog. At a small SFR, OFR is used to analyze the relationship between fluctuation extent and observation time.

As stated earlier, I, Q, U, and V can be observed from the polarimeter. In practice, the signal emitted from a pulsar and measured by a polarimeter can be expressed as a function of ϕ:

$$f(\phi)=I[1+d\cos(2\phi-2\beta)], \tag{6}$$

where I is the total intensity of incident X-ray photons, $d=Q/U$ is the degree of polarization, ϕ is the polarization position angle, and β is the inclination angle of the direction in which the polarized light can permeate on the polarimeter with respect to the principal axis. The PPA can be fitted by the polarization measurements of the pulsar. Fig. 2 shows the flowchart of attitude estimation using the polarization information of the pulsar.

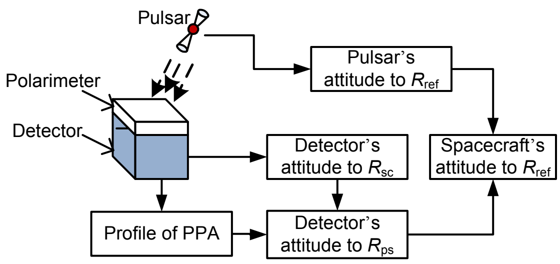

Fig. 2 The flowchart of attitude estimation using the polarization measurements of the pulsar

R_{ref}, an inertially fixed frame, denotes the celestial coordinates, with x axis pointing to Galactic North while the y axis in the direction of the first point of Aries. R_{ps}, an inertially fixed frame, denotes the pulsar coordinates, defined by the pulsar position in R_{ref}. The position is given by the right ascension (RA) of the ascending node and the declination of the pulsar. R_{sc}, a body fixed frame, denotes the spacecraft coordiantes, with the origin located at the center of mass of the spacecraft, x axis directed towards the principal axis of the detector, z axis directed along the rotation axis, and y axis completing a right-handed orthogonal frame

4 Numerical simulation and discussion

In the absence of other strong radiation sources, the observed PPA of pulsars can be fitted by Eq. (6). All the measurements are performed in an inertial frame. The profiles of linear polarization observations are expressed as a function of longitude. It is assumed that the periods of pulsars are already known, that the spacecraft velocity is a constant, and that the relativistic effects are negligible. The emission signals received by all polarimeters are synchronized. No cycle confusion exists. We analyze four pulsars here to check their stability of polarization and demonstrate the effectiveness of attitude estimation using polarization information. Table 1 shows the basic properties of the four candidate pulsars (Gould and Lyne, 1998; Sala *et al.*, 2004; European Pulsar Network, 2006).

The profiles of pulse (Lyne and Manchester, 1988; Gould and Lyne, 1998) and polarization are shown in Figs. 3a–3d. The measurement results of PPA are generated based on the data obtained from European Pulsar Network (2006). Position angles are plotted when the fractional linear polarization exceeds 15% and the uncertainty in PPA is less than 5°. The pulsars are chosen for their fractional mean linear polarization exceeding 20% and a wide distribution range in mean flux density and pulse period. After the pulse profiles and PPA curves are shown, two candidate pulsars are chosen as the navigation sources for the simulation of attitude estimation using their polarization information.

Table 1 The flux density, coordinates, period, and linearly polarized fraction of pulsars

Pulsar	S (mJy)	RA (°)	DEC (°)	Period (ms)	$<L>/S$
J0437-4715	149.3	4.62	−47.25	5.76	0.24
B0740-28	34.2	7.71	−28.38	166.75	0.61
B1919+21	56.5	19.21	21.53	1337.30	0.28
B2319+60	2.26	23.21	60.24	2256.48	0.23

S: mean flux density of the pulsar; RA: right ascension; DEC: declination; period: pulse period; $<L>/S$: fractional mean linear polarization, with $<L>$ being the flux density of mean linear polarization

From Figs. 3a–3d, it turns out that not all the PPAs are simply S-shaped. Due to the contamination from the space radiation environment, the shape of a polarization profile is not regular, but generally stable and constant. Based on the model of non-homogeneous Poisson distribution, simulated data is generated using the characteristics of two candidate pulsars, B0740-28 and B2319+60 (Table 1).

Monte-Carlo simulations are conducted to evaluate the proposed method. Fig. 4 shows the results of attitude estimation. Yaw angle is the major concern, since the other two attitude angles can be obtained using the aspect solution of the two pulsars. When SFR is less than 0.04, the range of yaw errors is within ±0.05°. In general, this accuracy level satisfies the requirements of most missions. When SFR is 0.06, the range of yaw errors is within ±0.12°. To achieve a better estimation precision, SFR should not exceed 0.04.

Fig. 5 shows the analysis results of SFR. Each line presents a variation tendency of SFR calculated with the observed profile and the standard profile. The observed profiles are obtained by accumulating the measurement values of 400, 300, 200, 100 time cycles at the longitude region of interest. The curves of SFR show that stability of PPA improves with an increase of radiation intensity. The shape of PPA has no obvious relation with the period, as well as the SFR. Under the same condition, high radiation

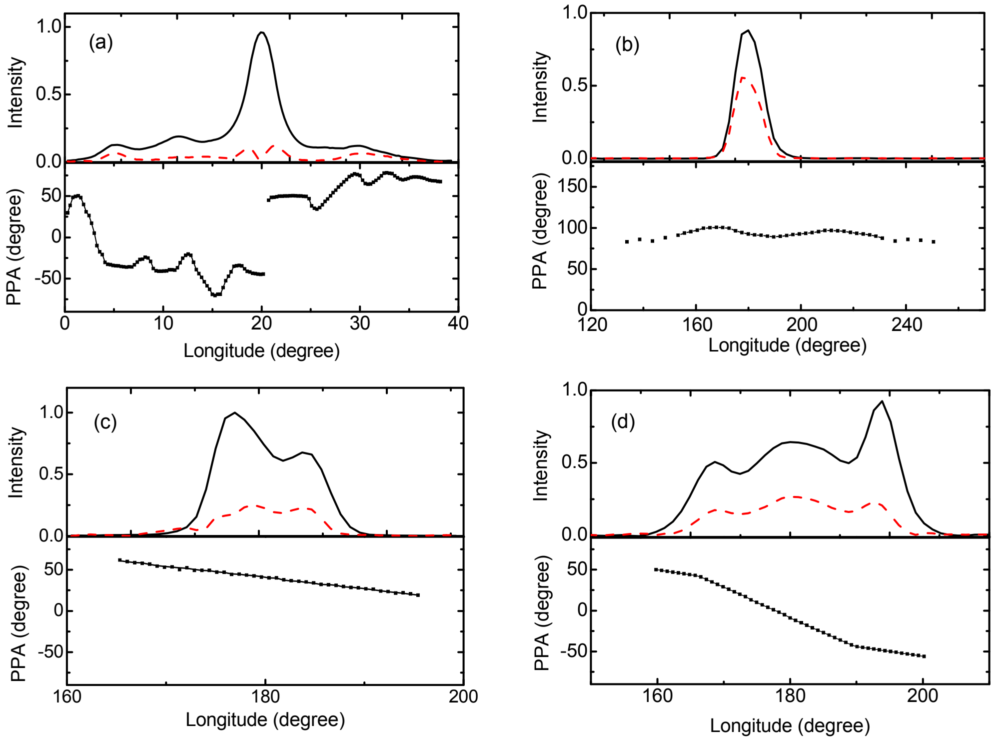

Fig. 3 Normalized total intensity (solid line) and linear polarization component (dashed line) of pulse and the polarization position angle (PPA)

(a) PSR J0437-4715 (610 MHz); (b) PSR B0740-28 (610 MHz); (c) PSR B1919+21 (610 MHz); (d) PSR B2319+60 (610 MHz)

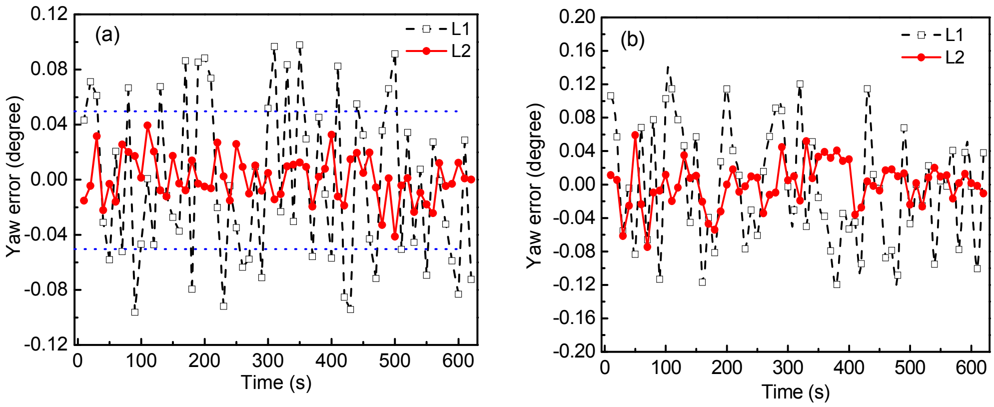

Fig. 4 Estimation errors of the yaw angle by PSR B0740-28 (a) and PSR B2319+60 (b) (L1: SFR=0.04; L2: SFR=0.06)

intensity will provide a better accuracy of measurement. Although the degree of polarization in the peak tends to be a low point, SFR at this longitude is very small; namely, the reliability and stability of the corresponding PPA is beneficial for applications of attitude determination.

SFR near the peak of the polarization profile reaches a minimal value as expected, where generally the attitude is estimated in practical applications. For both of these two pulsars analyzed, the longitude region including the minimal value of SFR corresponds to the region including the pulse peak in Figs. 3b and 3d. For PSR B0740-28, when observation duration is more than 100 cycles, SFR is smaller than 0.04 at the longitudes 173°−187°. However, to make SFR of PSR B2319+60 less than 0.04, more than 200 cycles are needed, and the usable longitude region is smaller than that of PSR B0740-28.

It is assumed that $P_{\mathrm{mean}}(\phi)$ has been obtained. Let n be 100. Fig. 6 presents the analysis results of OFR. Observation interval Δt represents the number of cycles used to obtain $P_i(\phi)$. Results are obtained at four longitudes approaching the pulse peak sequentially. With increase of Δt, OFR tends to decrease. In Fig. 6a, when the observation interval is larger than 150 cycles, OFR is smaller than 0.1; in Fig. 6b, OFR reaches the same level when the observation interval is larger than 200 cycles. In the case above, deviations of $P_i(\phi)$ and $P_{\mathrm{mean}}(\phi)$ are very small. Namely, all measurements $P_i(\phi)$ can be used for attitude estimation under this condition.

The SFR of two candidate pulsars can reach the specification of attitude estimation. By analyzing the OFR, it is further illustrated that given enough observation time, the profile of PPA is stable and effective at a region near the pulse peak, and Δt is inversely proportional to the flux intensity of the pulsar. As a whole, both pulsars can be used as guide stars to estimate spacecraft attitude using their polarization information.

5 Conclusions and future work

In summary, it is feasible to determine spacecraft attitude using pulsar polarization information. At least, it is enough as a tool to estimate the coarse attitude, according to the simulation results that the errors of the yaw angle are within a range of ±0.05°. Actually, the process of attitude determination via the polarization information of the pulsar is related mainly to the peak of the pulse profile. The higher the flux density of the pulsar, the more precise the PPA measured. Taking account of this, an analysis of the mean polarization profile should place more emphasis on the area near the profile peak. Based on effective

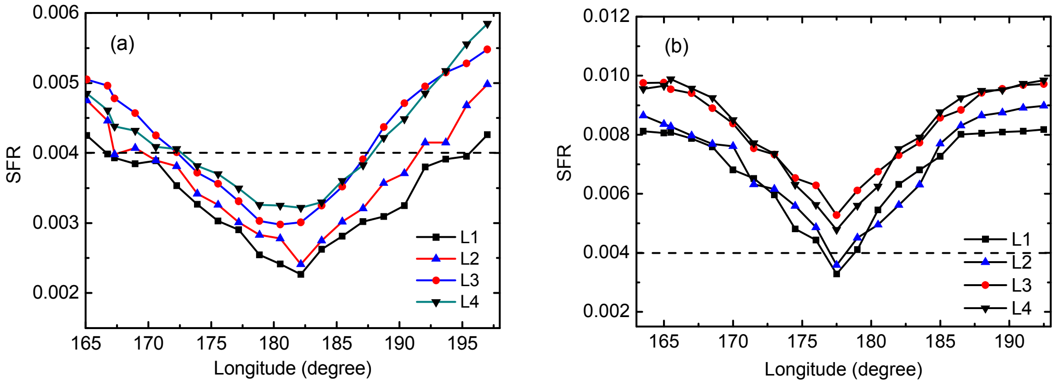

Fig. 5 Stability factors (SFR) of PSR B0740-28 (a) and PSR B2319+60 (b) with different numbers of cycles
L1, L2, L3, L4 correspond to 400, 300, 200, 100 cycles, respectively

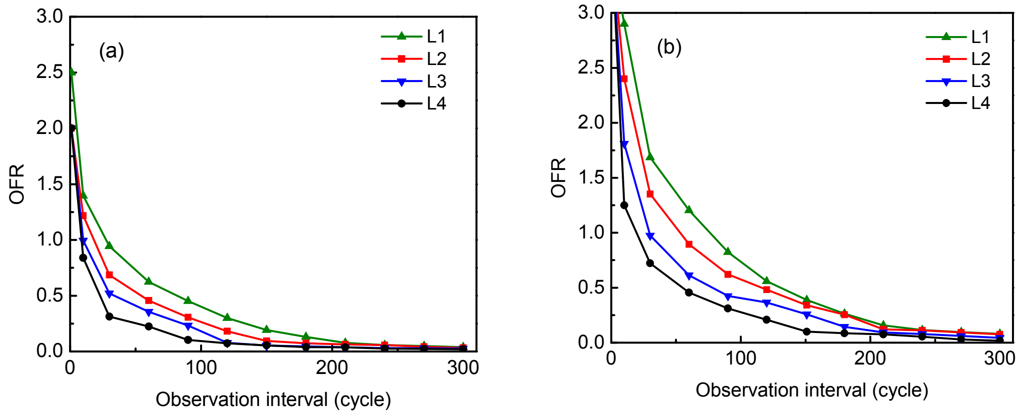

Fig. 6 Observation fluctuation factors (OFR) of PSR B0740-28 (a) and PSR B2319+60 (b)
In (a), L1, L2, L3, L4 correspond to longitudes 175°, 177°, 179°, 181°, respectively; in (b), L1, L2, L3, L4 correspond to longitudes 170°, 173°, 175°, 177°, respectively

polarization measurement, it is convenient to fuse information observed from the pulsar to determine spacecraft attitude. To better measure these characteristics, further studies on designing detectors with short response time, high resolution, and high sensitivity are necessary, as well as the data processing method. Moreover, phase variation of the polarization angle is not always an S-curve and cannot be modeled uniformly. Not all pulsars are reliable for attitude estimation. Thus, particular emphasis should be put on the issue of choosing reference pulsars to be used in future projects.

References

Bernhardt, M.G., Becher, W., Prinz, T., Breithuth, F.M., Walter, U., 2011. Autonomous Spacecraft Navigation Based on Pulsar Timing Information. 2nd ICST, p.1-4. [doi:10.1109/ICSpT.2011.6064649]

Blaskiewicz, M., Cordes, J.M., Wasserman, I., 1991. A relativistic model of pulsar polarization. *ApJ*, **370**:643-669. [doi:10.1086/169850]

Camilo, F., Reynolds, J., Johnston, S., Halpern, J.P., Ransom, S.M., van Straten, W., 2007. Polarized radio emission from the magnetar XTE J1810-197. *ApJ*, **659**(1):L37-L40. [doi:10.1086/516630]

Chu, J.K., Zhao, K.C., Zhang, Q., Wang, T.C., 2008. Construction and performance test of a novel polarization sensor for navigation. *Sens. Actuat. A*, **148**(1):75-82. [doi:10.1016/j.sna.2008.07.016]

Dean, A.J., Clark, D.J., Stephen, J.B., McBride, V.A., Bassano, A., Bird, A.J., Hill, A.B., Shaw, S.E., Ubertini, P., 2008. Polarized gamma-ray emission from the Crab. *Science*, **321**(5893):1183-1185. [doi:10.1126/science.1149056]

European Pulsar Network, 2006. Citing Electronic Sources of Information. Available from http://www.jb.man.ac.uk/~pulsar/Resources/epn/browser.html [Accessed on May 22, 2012].

Forot, M., Laurent, P.L., Grenier, I.A., Gouiffès, C., Lebrun, F., 2008. Polarization of the Crab pulsar and nebula as observed by the INTEGRAL/IBIS telescope. *ApJ*, **688**(1): L29-L32. [doi:10.1086/593974]

Gould, D.M., Lyne, A.G., 1998. Multifrequency polarimetry of 300 radio pulsars. *Mon. Not. R. Astr. Soc.*, **301**(1): 235-260. [doi:10.1046/j.1365-8711.1998.02018.x]

Han, J.L., Demorest, P.B., van Straten, W., Lyne, A.G., 2009. Polarization observations of 100 pulsars at 774 MHz by the GREEN BANK telescope. *ApJS*, **181**(2):557-571. [doi:10.1088/0067-0049/181/2/557]

Horváth, G., Varju, D., 1963. Polarized Light in Animal Vision: Polarization Patterns in Nature. Springer, Heidelberg, Germany, p.2-25.

Hughes, J.P., Long, K.S., Novick, R., 1984. A search for X-ray polarization in comic X-ray sources. *ApJ*, **280**: 255-258. [doi:10.1086/161992]

Johnston, S., Hobbs, G., Vigeland, S., Kramer, M., Weisberg, J.M., Lyne, A.G., 2005. Evidence for alignment of the rotation and vectors in pulsars. *Mon. Not. R. Astr. Soc.*, **364**(4):1397-1412. [doi:10.1111/j.1365-2966.2005.09669.x]

Jones, D.H.P., Smith, F.G., Wallace, P.T., 1981. Linear polarization of optical radiation from the Crab pulsar. *Mon. Not. R. Astr. Soc.*, **196**:943-953.

Karastergiou, A., Johnston, S., 2007. An empirical model for the beams of radio pulsars. *Mon. Not. R. Astr. Soc.*, **380**(4):1678-1684. [doi:10.1111/j.1365-2966.2007.12237.x]

Krawczynski, H., Garson, A.III, Guo, Q., Baring, M.G., Ghosh, P., Beilicke, M., Lee, K., 2011. Scientific prospects for hard X-ray polarimetry. *Astr. Phys.*, **34**(7):550-567. [doi:10.1016/j.astropartphys.2010.12.001]

Labhart, T., 1988. Polarization-opponent interneurons in the insect visual system. *Nature*, **331**(6155):435-437. [doi:10.1038/331435a0]

Lyne, A.G., Manchester, R.N., 1988. The shape of pulsar radio beams. *J. Mon. Not. R. Astr. Soc.*, **234**:477-508.

Manchester, R.N., Johnston, S., 1995. Polarization properties of two pulsars. *ApJ*, **441**(2):L65-L68. [doi:10.1086/187791]

Manchester, R.N., Taylor, J.H., 1977. Pulsars. WH Freeman, San Francisco, CA, USA, p.84-90.

McKinnon, M.M., Stinebring, D.R., 1998. A statistical model for the orthogonal modes of polarization in pulsar radio emission. *ApJ*, **502**(2):883-897. [doi:10.1086/305924]

McKinnon, M.M., Stinebring, D.R., 2000. The mode-separated pulse profiles of pulsar radio emission. *ApJ*, **529**(1):435-446. [doi:10.1086/308264]

McMaster, W.H., 1954. Polarization and the Stokes parameters. *Am. J. Phys.*, **22**(6):351. [doi:10.1119/1.1933744]

Mizuno, T., Arimoto, M., Axelsson, M., Bjornsson, C.I., Bogaert, G., Carlson, P., Craig, W., Fukazawa, Y., Gunji, S., Hjalmarsdotter, L., *et al.*, 2007. High Sensitivity Balloon-Borne Hard X-Ray/Soft Gamma-Ray Polarimeter PoGOLite. IEEE Nuclear Science Symp. Conf. Record, **4**:2538-2544. [doi:10.1109/NSSMIC.2007.4436669]

Morris, D., Graham, D.A., Sieber, W., Bartel, N., Thomasson, P., 1981. Observations of the polarization of average pulsar profiles at high frequency. *Astron. Astrophys. Suppl. Ser.*, **46**:421-472.

Pearce, M., Florén, H.G., Jackson, M., Kamae, T., Kiss, M., Kole, M., Moretti, E., Olofsson, G., Rydström, S., Strömberg, J.E., *et al.*, 2012. Balloon-Borne Hard X-Ray Polarimetry with PoGOLite. arXiv preprint, arXiv:1211.5094.

Petrova, S.A., 2001. On the origin of orthogonal polarization modes in pulsar radio emission. *A&A*, **378**(3):883-897. [doi:10.1051/0004-6361:20011297]

Radhakrishnan, V., Cooke, D.J., 1969. Magnetic poles and the polarization structure of pulsar radiation. *A&A*, **3**:225.

Sala, J., Urruela, A., Villares, X., Estalella, R., Paredes, J.M., 2004. Feasibility Study for a Spacecraft Navigation System Relying on Pulsar Timing Information. Report No.

ARIADNA Study 03/4202, Department of Signal Theory and Communications, Universitat Politècnica de Catalunya, Spain.
Silver, E.H., Kestenbaum, H.L., Long, K.S., Novick, R., Wolff, R.S., Weisskopf, M.C., 1978. Search for X-ray polarization in the Crab pulsar. *ApJ*, **225**:221-225. [doi:10.1086/156485]
Slowikowska, A., Kanbach, G., Kramer, M., Stefanescu, A., 2009. Optical polarization of the Crab pulsar: precision measurements and comparison to the radio emission. *Mon. Not. R. Astr. Soc.*, **397**(1):103-123. [doi:10.1111/j.1365-2966.2009.14935.x]
Smith, F.G., Jones, D.H.P., Dick, J.S.B., Pike, C.D., 1988. The optical polarization of the Crab pulsar. *Mon. Not. R. Astr. Soc.*, **233**(2):305-319.
Weisskopf, M.C., Silver, E.H., Kestenbaum, H.L., Long, K.S., Novick, R., 1978. A precision measurement of the X-ray polarization of the Crab nebula without pulsar contamination. *ApJ*, **220**:L117-L121. [doi:10.1086/182648]
Zhao, K.C., Chu, J.K., Wang, T.C., Zhang, Q., 2009. A novel angle algorithm of polarization sensor for navigation. *IEEE Trans. Instrum. Meas.*, **58**(8):2791-2796. [doi:10.1109/TIM.2009.2016299]