



A frequency domain design of PID controller for an AVR system

Md Nishat ANWAR, Somnath PAN

(Department of Electrical Engineering, Indian School of Mines, Dhanbad 826004, India)

E-mail: nishatnith@gmail.com; somnath_pan@hotmail.com

Received Aug. 15, 2013; Revision accepted Dec. 18, 2013; Crosschecked Mar. 17, 2014

Abstract: We propose a new proportional-integral-derivative (PID) controller design method for an automatic voltage regulation (AVR) system based on approximate model matching in the frequency domain. The parameters of the PID controller are obtained by approximate frequency response matching between the closed-loop control system and a reference model with the desired specifications. Two low frequency points are required for matching the frequency response, and the design method yields linear algebraic equations, solution of which gives the controller parameters. The effectiveness of the proposed method is demonstrated through examples taken from the literature and comparison with some popular methods.

Key words: Automatic voltage regulation (AVR), PID controller, Frequency response matching

doi:10.1631/jzus.C1300218

Document code: A

CLC number: TP273

1 Introduction

The proportional-integral-derivative (PID) controller with its variants is the most widely accepted controller for industrial applications due to its simple structure and ease of application. It provides solutions to a wide range of processes by improving transient performance and steady state performance. The performance of the desired control system depends on the plant dynamics as well as the gains of the PID controller. Researchers have presented various tuning methods for PID controllers and these may be found in Astrom and Hagglund (1995) and O'Dwyer (2006). The various design techniques such as the frequency domain design techniques (Ziegler and Nichols, 1942; Wang *et al.*, 1995), internal model control (IMC) (Rivera *et al.*, 1986; Tan, 2010), optimization of the error criteria (Panagopoulos *et al.*, 2002; Shen, 2002), direct synthesis design (Chen and Seborg, 2002), and gain- and phase-margin based design (Ho *et al.*, 1995) provide a wide range of choices for the selection of PID parameters.

The automatic voltage regulation (AVR) systems continuously adjust the field excitation to maintain the generator terminal voltage at a specified level (Kundur, 1994). The PID controller design method for an AVR system using particle swarm optimization (PSO) was presented in Gaing (2004), one using a chaotic ant swarm (CAS) based algorithm was presented in Zhu *et al.* (2009), and an intelligent particle swarm optimized fuzzy PID controller was presented in Mukherjee and Ghoshal (2007). Recently, fractional order PID controllers for AVR systems have been getting increased attention. Zamani *et al.* (2009) proposed a fractional order PID based on PSO with a novel cost function which considers time and frequency domain specifications. Pan and Das (2013) proposed a fractional order PID controller based on a non-dominated sorting genetic algorithm (NSGA) with multi-objective optimization which ensures stability and robustness. A model reference adaptive control (MRAC) with a genetic algorithm (GA) optimized fractional order PID controller has been proposed by Aguila-Camacho and Duarte-Mermoud (2013).

In this paper, a new PID controller design method based on frequency response matching for AVR

systems is proposed. For the PID controller design, the desired requirement of the overall system is first translated into a reference model and then the overall system transfer function including the unknown controller is matched with the reference model in the frequency domain, giving the desired controller of the high order dynamics, which is further simplified to a PID controller by approximate frequency response matching using linear algebraic equations.

2 AVR system

The PID controller improves the steady state as well as the dynamic response of the system, and is implemented in parallel (Ang et al., 2005) to regulate the voltage and reactive power of a synchronous generator, as given by

$$C(s) = K_p + \frac{K_I}{s} + K_D s, \quad (1)$$

where K_p , K_I , and K_D are the proportional, integral, and derivative constants, respectively, of the controller employed in the control architecture (Fig. 1).

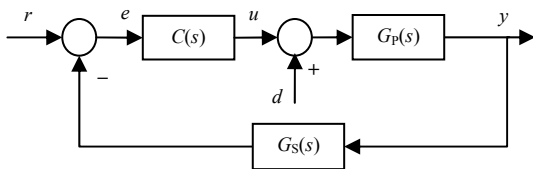


Fig. 1 Unity negative output feedback configuration r is the input, e is the error, u is the controller output, d is the disturbance, y is the output to the plant, and $G_p(s)$ and $G_s(s)$ are the linearized model blocks of an AVR system

The AVR system maintains the terminal voltage magnitude of an alternator to a specified value by regulating the reactive power flow by controlling the generator excitation. Typical components of an AVR system (Kundur, 1994) are amplifier, exciter, generator, and sensor. The linearized models of various components are shown in Fig. 2. The parameters of the AVR model and the transfer function of each item are shown in Table 1 (Gaing 2004).

Table 1 The parameters of an AVR system

Component	Transfer function	Parameter limits
Amplifier	$G_A(s) = \frac{V_R(s)}{V_u(s)} = \frac{K_A}{1 + \tau_A s}$	$10 \leq K_A \leq 400$, $0.02 \text{ s} \leq \tau_A \leq 0.1 \text{ s}$
Exciter	$G_E(s) = \frac{V_F(s)}{V_R(s)} = \frac{K_E}{1 + \tau_E s}$	$1 \leq K_E \leq 10$, $0.4 \text{ s} \leq \tau_E \leq 1.0 \text{ s}$
Generator	$G_G(s) = \frac{V_t(s)}{V_F(s)} = \frac{K_G}{1 + \tau_G s}$	K_G depends on load (0.7–1.0), $1.0 \text{ s} \leq \tau_G \leq 2.0 \text{ s}$
Sensor	$G_s(s) = \frac{V_s(s)}{V_t(s)} = \frac{K_s}{1 + \tau_s s}$	$0.001 \text{ s} \leq \tau_s \leq 0.06 \text{ s}$ $K_s = 1$

3 Design method

The aim of an AVR system is to keep the terminal voltage of the generator at a steady value, which is affected mainly by the reactive power flow. The generator reactive power is controlled by the field excitation system. The problem may be stated as finding the parameters of the PID controller in the control system configuration of Fig. 1 or Fig. 2, so as to keep the terminal voltage $V_t(s)$ at a specified value $V_{ref}(s)$.

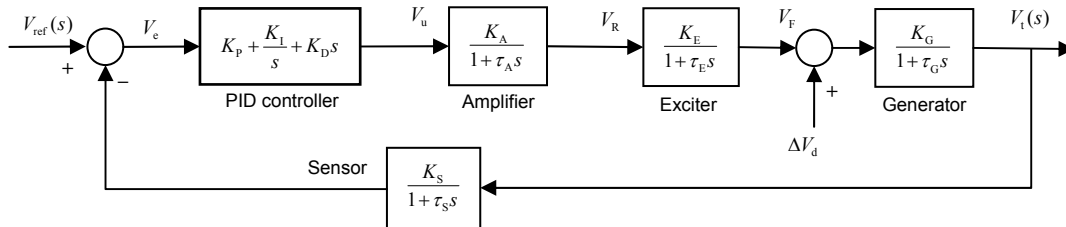


Fig. 2 Control configuration of an AVR system with a PID controller

$V_t(s)$ is the terminal voltage, $V_{ref}(s)$ is the reference voltage to be achieved by the AVR system, and ΔV_d is the disturbance due to the change of reactive power demand

From Fig. 2, the transfer function of the closed-loop system from r to y may be written as

$$G_{r,y}(s) = \frac{V_i(s)}{V_{ref}(s)} = \frac{C(s)G_p(s)}{1 + C(s)G_p(s)F(s)}. \quad (2)$$

From the equivalence of Figs. 1 and 2,

$$G_p(s) = G_A(s)G_E(s)G_G(s) = \frac{K_A K_E K_G}{(1 + \tau_A s)(1 + \tau_E s)(1 + \tau_G s)}, \quad (3)$$

and

$$G_S(s) = K_S / (1 + \tau_S s). \quad (4)$$

The first step of the design procedure is to choose or construct a suitable reference model $M_{r,y}(s)$ which incorporates the desired specification of the overall control system from r to y .

The reference model is chosen as

$$M_{r,y}(s) = \frac{P(s)}{Q(s)}, \quad (5)$$

where $P(s)/Q(s)$ is a rational polynomial.

To achieve the desired set-point response of the closed-loop system, the frequency response of $G_{r,y}(s)$ and that of $M_{r,y}(s)$ are matched:

$$G_{r,y}(s) \Big|_{s=j\omega} = \frac{C(s)G_p(s)}{1 + C(s)G_p(s)F(s)} \Big|_{s=j\omega} \cong M_{r,y}(s) \Big|_{s=j\omega}, \quad (6)$$

where the left-hand side expression is equivalent with the right-hand side expression in terms of frequency response, and the unknown parameters of controller $C(s)$ are to be evaluated. Eq. (6) may be written as

$$\begin{aligned} C(s) \Big|_{s=j\omega} &\cong \frac{M_{r,y}(s)}{G_p(s)[1 - M_{r,y}(s)F(s)]} \Big|_{s=j\omega} \\ &= \frac{P(s)(1 + \tau_A s)(1 + \tau_E s)(1 + \tau_G s)(1 + \tau_S s)}{K_A K_E K_G [Q(s)(1 + \tau_S s) - P(s)K_S]} \Big|_{s=j\omega} \\ &= H(s) \Big|_{s=j\omega}. \end{aligned} \quad (7)$$

Here, $H(s)$ is the controller that would satisfy the design requirements ideally, where the order and structure of $H(s)$ may not be suitable for implementation. However, Eq. (7) may be written as

$$C_R(\omega) + jC_I(\omega) \cong H_R(\omega) + jH_I(\omega), \quad (8)$$

where

$$\begin{aligned} C(s) \Big|_{s=j\omega} &= C_R(\omega) + jC_I(\omega), \\ H(s) \Big|_{s=j\omega} &= H_R(\omega) + jH_I(\omega), \end{aligned}$$

and $C_R(\omega)$, $C_I(\omega)$, $H_R(\omega)$, $H_I(\omega)$ are the real functions of ω . Separating the real and imaginary parts, Eq. (8) may be written as

$$C_R(\omega) \cong H_R(\omega), \quad C_I(\omega) \cong H_I(\omega). \quad (9)$$

To force the equivalence of two real functions, $C_R(\omega)$ and $C_I(\omega)$ with $H_R(\omega)$ and $H_I(\omega)$ respectively, one may equate the appropriate number of initial terms of the corresponding Taylor series expansion around $\omega=0$. Thus, to accomplish an approximate matching of the left-hand side functions in Eq. (9) with the corresponding functions on the right-hand side, the initial N derivatives of the corresponding functions are equated at $\omega=0$ to give

$$\frac{d^k}{d\omega^k} C_R(\omega) \Big|_{\omega=0} = \frac{d^k}{d\omega^k} H_R(\omega) \Big|_{\omega=0}, \quad (10)$$

$$\frac{d^k}{d\omega^k} C_I(\omega) \Big|_{\omega=0} = \frac{d^k}{d\omega^k} H_I(\omega) \Big|_{\omega=0}. \quad (11)$$

Now, using the divided difference calculus as in Pan and Pal (1995), it may be observed that the derivative relation in Eq. (10) will be satisfied approximately, if the following algebraic relation is satisfied:

$$C_R(\omega) \Big|_{\omega=\omega_k} = H_R(\omega) \Big|_{\omega=\omega_k}, \quad k \in [0, N-1], \quad (12)$$

where ω_k are small positive values around $\omega=0$. Similarly,

$$C_I(\omega) \Big|_{\omega=\omega_k} = H_I(\omega) \Big|_{\omega=\omega_k}, \quad k \in [0, N-1]. \quad (13)$$

Now, theoretically, the range of ω is from 0 to ∞ and for such an infinite range, it is meaningless to find ω_k values which are small. The smallness of the frequency values chosen is made consistent with the consideration of the effective range of the frequency response. This is elaborated as follows: If τ is the dominant time constant of the plant, 10 times $2\pi/\tau$, which is $20\pi/\tau$, can be assumed to be the effective range of the dominant frequency response of the plant. Hence, the low frequency values, for the purpose of matching, may be selected around 0.01 times the effective range. Such frequency points for matching give good results for most plants. It is clear from Eqs. (12) and (13) that N values of ω give $2N$ linear algebraic equations with the known parameters. For three unknowns of the PID controller, N is at least equal to two. For two frequency points ω_0 and ω_1 , the following expression is obtained:

$$A\bar{x} = \bar{b}, \tag{14}$$

where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/\omega_0 & \omega_0 \\ 1 & 0 & 0 \\ 0 & -1/\omega_1 & \omega_1 \end{bmatrix}, \quad \bar{x} = [K_P \quad K_I \quad K_D]^T,$$

$$\bar{b} = [H_R(\omega_0) \quad H_I(\omega_0) \quad H_R(\omega_1) \quad H_I(\omega_1)]^T.$$

Directly from Eq. (14), we obtain two values of K_P as $K_{P1}=H_R(\omega_0)$, $K_{P2}=H_R(\omega_1)$.

It is observed from various examples that $K_{P1} \approx K_{P2}$. Thus, we may take the value of K_P as any one of K_{P1} and K_{P2} or an average of these.

Then, to evaluate the remaining parameters, K_I and K_D , Eq. (14) may be simplified as

$$A_1\bar{x}_1 = \bar{b}_1, \tag{15}$$

where

$$A_1 = \begin{bmatrix} -1/\omega_0 & \omega_0 \\ -1/\omega_1 & \omega_1 \end{bmatrix}, \quad \bar{x}_1 = [K_I \quad K_D]^T,$$

$$\bar{b}_1 = [H_I(\omega_0) \quad H_I(\omega_1)]^T.$$

Then, the solution of Eq. (15) determines K_I and K_D . Thus, the parameters of the PID controller are evaluated.

4 Simulation results

Example 1 An AVR system taken from Gaing (2004) is considered with the parameters given in Table 2.

Table 2 Parameters used for Example 1

Component	Gain	Time constant (s)
Amplifier	$K_A=10$	$\tau_A=0.1$
Exciter	$K_E=1$	$\tau_E=0.4$
Generator	$K_G=1$	$\tau_G=1.0$
Sensor	$K_S=1$	$\tau_S=0.01$

First, the nominal system is simulated without the controller. The terminal voltage response with unit step input is shown in Fig. 3, where it is observed that the peak overshoot is $M_p=50.6\%$, the steady state error is $e_{ss}=9.09\%$, the rise time is $t_r=0.269$ s, and the settling time is $t_s=6.98$ s.

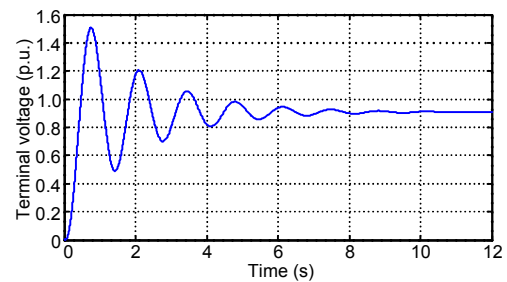


Fig. 3 Generator terminal voltage of an AVR system without a controller

The reference model is chosen as

$$M_{r,y}(s) = \frac{1}{0.22s + 1},$$

which gives the settling time as 0.86 s and the rise time as 0.48 s. The frequency points are chosen as $\omega_0=0.01$ rad/s and $\omega_1=0.02$ rad/s for frequency response matching, and the design procedure yields the PID controller as

$$C(s) = 0.652 + 0.434/s + 0.236s.$$

Figs. 4 and 5 and Table 3 show the set-point responses due to the unit step input at $t=0$ and responses due to unit step disturbance ΔV_d at $t=5$ s along with the controller outputs by the proposed controller as well as by those of Gaing (2004),

Mukherjee and Ghoshal (2007), and Kim (2011). The overshoot in the set-point response by the proposed method is less than those given by Gaing (2004) and Kim (2011)'s methods, while the settling time by the proposed method is less than those given by Mukherjee and Ghoshal (2007) and Kim (2011)'s methods. Table 3 shows the stability comparison in terms of gain margin (GM) and phase margin (PM).

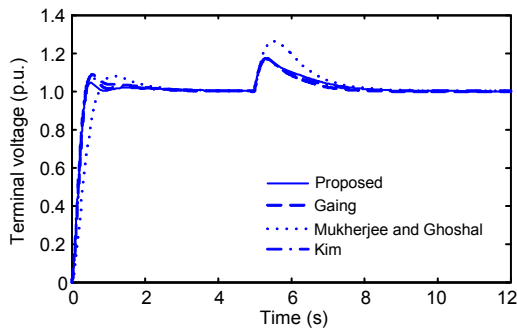


Fig. 4 Terminal voltage of the generator for Example 1

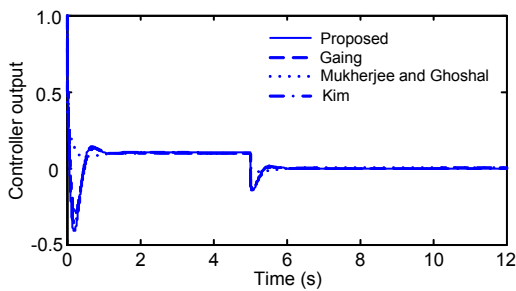


Fig. 5 Controller output for Example 1

To show the robustness of the proposed controller, +50% changes in the gain and time constant of the generator are considered. The corresponding response is shown in Fig. 6, and performances of various methods are given in Table 4. The overall performance of the proposed method is favorable compared with those of the other methods. For the proposed controller, the maximum output due to disturbance is the least and the overall integral square error (ISE) is the lowest.

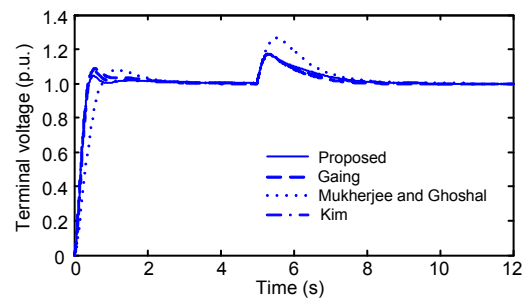


Fig. 6 Terminal voltage of the generator for Example 1 with +50% changes in K_G and τ_G

Example 2 An AVR system taken from Zhu *et al.* (2009) is considered. The parameters are given in Table 5.

The reference model $M_{r,y}(s)$ is chosen as

$$M_{r,y}(s) = \frac{1}{0.25s + 1}$$

Table 3 Performance comparison for Example 1

Method	K_P	K_I	K_D	Set-point response		GM	PM	Load-disturbance response		ISE*
				M_P (%)	t_s (s)			M_P (%)	t_s (s)	
Proposed	0.652	0.434	0.236	2.0	0.41	26.1	67.7	16.1	2.75	0.168
Gaing (2004)	0.657	0.538	0.218	4.5	0.40	25.8	67.6	16.6	2.15	0.171
Mukherjee and Ghoshal (2007)	0.374	0.268	0.100	0	0.92	32.6	71.9	24.5	3.20	0.307
Kim (2011)	0.672	0.478	0.229	3.7	1.06	26.2	66.4	16.3	2.50	0.178

GM: gain margin; PM: phase margin. ISE: integral square error (* for 12 s)

Table 4 Performance comparison for +50% changes in K_G and τ_G for Example 1

Method	K_P	K_I	K_D	Set-point response		Load-disturbance response		ISE*
				M_P (%)	t_s (s)	M_P (%)	t_s (s)	
Proposed	0.652	0.434	0.236	4.5	0.70	17.0	2.61	0.169
Gaing (2004)	0.657	0.538	0.218	8.6	2.04	17.4	2.05	0.173
Mukherjee and Ghoshal (2007)	0.374	0.268	0.100	8.0	2.30	26.0	2.90	0.309
Kim (2011)	0.672	0.478	0.229	7.5	1.63	17.1	2.40	0.168

ISE: integral square error (* for 10 s)

Table 5 Parameters used for Example 2

Component	Gain	Time constant (s)
Amplifier	$K_A=12$	$\tau_A=0.09$
Exciter	$K_E=10$	$\tau_E=0.5$
Generator	$K_G=0.1$	$\tau_G=1.1$
Sensor	$K_S=1$	$\tau_S=0.02$

which gives settling time as 0.97 s and rise time as 0.55 s. The low frequency points are selected as $\omega_0=0.01$ rad/s and $\omega_1=0.02$ rad/s for the purpose of frequency response matching. The obtained PID controller is as given below:

$$C(s) = 0.5093 + 0.33/s + 0.20s.$$

Figs. 7 and 8 show the unit step response of the terminal voltage and the controller output, respectively, using the proposed controller and that given by Zhu *et al.* (2009), and Table 6 tabulates the performance comparison. A unit step disturbance is applied at $t=3$ s and the performance comparison is shown in Figs. 7 and 8 and Table 6. The peak overshoot by the proposed method is less than that given by Zhu *et al.* (2009). For comparing the stability, GM and PM have been shown in Table 6. The robustness of the proposed controller is studied by considering +50% changes in the gain and time constant of the generator. The corresponding response is shown in Fig. 9, and the performances are given in Table 7. In terms of overall performance, the proposed controller is comparable with that of Zhu *et al.* (2009).

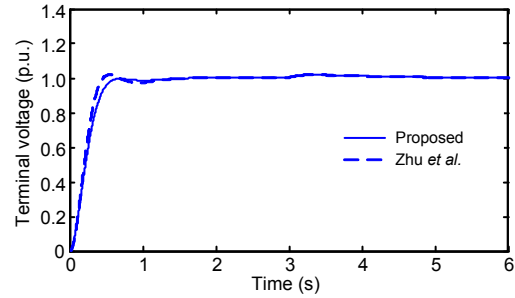


Fig. 7 Terminal voltage of the generator for Example 2

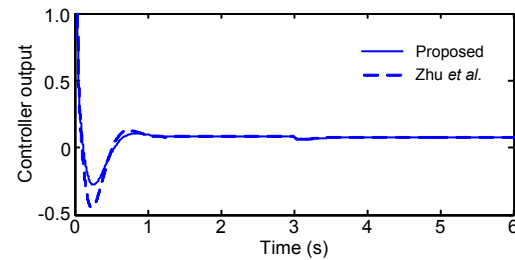


Fig. 8 Controller output for Example 2

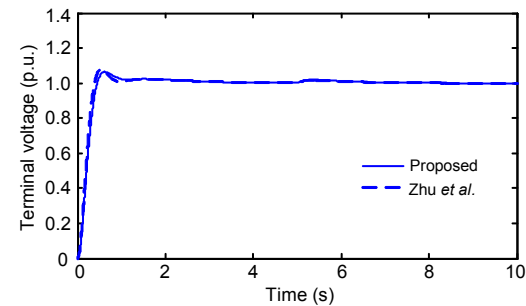


Fig. 9 Terminal voltage of the generator for Example 2 with +50% changes in K_G and τ_G

Table 6 Performance comparison for Example 2

Method	K_P	K_I	K_D	Set-point response		GM	PM	Load-disturbance response		ISE*
				M_P (%)	t_s (s)			M_P (%)	t_s (s)	
Proposed	0.5093	0.3300	0.2000	0.4	0.51	22.6	70.5	2.0	1.4	0.182
Zhu <i>et al.</i> (2009)	0.5613	0.3670	0.2326	2.0	0.42	21.4	68.0	2.0	1.4	0.167

GM: gain margin; PM: phase margin. ISE: integral square error (* for 6 s)

Table 7 Performance comparison for +50% changes in K_G and τ_G for Example 2

Method	K_P	K_I	K_D	Set-point response		Load-disturbance response		ISE*
				M_P (%)	t_s (s)	M_P (%)	t_s (s)	
Proposed	0.5093	0.3300	0.2000	6.4	1.90	2.0	0.40	0.181
Zhu <i>et al.</i> (2009)	0.5613	0.3670	0.2326	7.8	1.83	2.0	0.35	0.164

ISE: integral square error (* for 6 s)

5 Conclusions

A new frequency domain model matching method has been described for the design of a PID controller for an AVR system. The method has been compared favorably with some popular design methods through examples taken from the literature. The frequency response matching between the reference model and the AVR system to be designed is done at two low frequency points to arrive at a set of linear algebraic equations, whose solution gives the controller parameters. The method does not require any elaborate frequency response analysis or mathematically involved optimization technique. In summary, the method is mathematically simple and its computational burden is very small; thus, it gives a PID controller of good overall performance.

References

- Aguila-Camacho, N., Duarte-Mermoud, M.A., 2013. Fractional adaptive control for an automatic voltage regulator. *ISA Trans.*, **52**(6):807-815. [doi:10.1016/j.isatra.2013.06.005]
- Ang, K.H., Chong, G., Li, Y., 2005. PID control system analysis, design and technology. *IEEE Trans. Contr. Syst. Technol.*, **13**(4):559-576. [doi:10.1109/TCST.2005.847331]
- Astrom, K.J., Hagglund, T., 1995. PID Controllers Theory Design and Tuning (2nd Ed.). Instrument Society of America, Research Triangle Park, North Carolina.
- Chen, D., Seborg, D.E., 2002. PI/PID controller design based on direct synthesis and disturbance rejection. *Ind. Eng. Chem. Res.*, **41**(19):4807-4822. [doi:10.1021/ie010756m]
- Gaing, Z.L., 2004. A particle swarm optimization approach for optimum design of PID controller in AVR system. *IEEE Trans. Energy Conv.*, **19**(2):384-391. [doi:10.1109/TEC.2003.821821]
- Ho, W.K., Hang, C.C., Cao, L.S., 1995. Tuning of PID controllers based on gain and phase margin specification. *Automatica*, **31**(3):497-502. [doi:10.1016/0005-1098(94)00130-B]
- Kim, D.H., 2011. Hybrid GA-BF based intelligent PID controller tuning for AVR system. *Appl. Soft Comput.*, **11**(1):11-22. [doi:10.1016/j.asoc.2009.01.004]
- Kundur, P., 1994. Power System Stability and Control. McGraw Hill, New York.
- Mukherjee, V., Ghoshal, S.P., 2007. Intelligent particle swarm optimized fuzzy PID controller for AVR system. *Electr. Power Syst. Res.*, **77**(12):1689-1698. [doi:10.1016/j.epsr.2006.12.004]
- O'Dwyer, A., 2006. Handbook of PI and PID Controller Tuning Rules (2nd Ed.). Imperial College Press, London. [doi:10.1142/p424]
- Pan, I., Das, S., 2013. Frequency domain design of fractional order PID controller for AVR system using chaotic multi-objective optimization. *Int. J. Electr. Power Energy Syst.*, **51**:106-118. [doi:10.1016/j.ijepes.2013.02.021]
- Pan, S., Pal, J., 1995. Reduced order modelling of discrete-time systems. *Appl. Math. Model.*, **19**(3):133-138. [doi:10.1016/0307-904X(94)00010-4]
- Panagopoulos, H., Astrom, K.J., Hagglund, T., 2002. Design of PID controllers based on constrained optimisation. *IEE Proc.-Contr. Theory Appl.*, **149**(1):32-40. [doi:10.1049/ip-cta:20020102]
- Rivera, D.E., Morari, M., Skogestad, S., 1986. Internal model control. 4. PID controller design. *Ind. Eng. Chem. Process Des. Dev.*, **25**(1):252-265. [doi:10.1021/i200032a041]
- Shen, J.C., 2002. New tuning method for PID controller. *ISA Trans.*, **41**(4):473-484. [doi:10.1016/S0019-0578(07)60103-7]
- Tan, W., 2010. Unified tuning of PID load frequency controller for power system via IMC. *IEEE Trans. Power Syst.*, **25**(1):341-350. [doi:10.1109/TPWRS.2009.2036463]
- Wang, L., Barnes, T.J.D., Cluett, W.R., 1995. New frequency domain design method for PID controllers. *IEE Proc.-Contr. Theory Appl.*, **142**(4):265-271. [doi:10.1049/ip-cta:19951859]
- Zamani, M., Karimi-Ghartemani, M., Parniani, M., 2009. Design of a fractional order PID controller for an AVR using particle swarm optimization. *Contr. Eng. Pract.*, **17**(12):1380-1387. [doi:10.1016/j.conengprac.2009.07.005]
- Zhu, H., Li, L., Zhao, Y., et al., 2009. CAS algorithm-based optimum design of PID controller in AVR system. *Chaos Sol. Fract.*, **42**(2):792-800. [doi:10.1016/j.chaos.2009.02.006]
- Ziegler, J.G., Nichols, N.B., 1942. Optimum settings for automatic controllers. *Trans. ASME*, **64**:759-768.