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Knowledge modeling based on interval-valued fuzzy rough set and similarity inference: prediction of welding distortion*#

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Abstract: Knowledge-based modeling is a trend in complex system modeling technology. To extract the process knowledge from an information system, an approach of knowledge modeling based on interval-valued fuzzy rough set is presented in this paper, in which attribute reduction is a key to obtain the simplified knowledge model. Through defining dependency and inclusion functions, algorithms for attribute reduction and rule extraction are obtained. The approximation inference plays an important role in the development of the fuzzy system. To improve the inference mechanism, we provide a method of similarity-based inference in an interval-valued fuzzy environment. Combining the conventional compositional rule of inference with similarity based approximate reasoning, an inference result is deduced via rule translation, similarity matching, relation modification, and projection operation. This approach is applied to the problem of predicting welding distortion in marine structures, and the experimental results validate the effectiveness of the proposed methods of knowledge modeling and similarity-based inference.

Key words: Knowledge modeling, Interval-valued fuzzy rough set, Similarity-based inference, Welding distortion prediction
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1 Introduction

In the past decades, conventional modeling methods have made great achievements in many fields like manufacturing, chemical engineering, and medical psychology. Nevertheless, in the real world most systems are too complicated to be described by precise mathematical models. Therefore, knowledge-based modeling has recently become a popular form of system modeling. Since a fuzzy model will approximate to any complicated nonlinear function sufficiently, fuzzy-based modeling is considered an ideal approach for complicated system modeling.

In the process of establishing a fuzzy inference system, it is often hard to obtain rules by manual means due to the complexity of a system, which partly restricts the application of fuzzy logic techniques. For a multi-variable fuzzy system, the size of rules may increase geometrically, causing a so-called combinational explosion of the rule base. Thus, to obtain a simplified fuzzy rule set, it is necessary to reduce the size of the variables and linguistic values in a system. As a mathematical tool to handle incomplete and imprecise data, rough set theory (RS) proposed by Pawlak (1982) has been extensively applied in data mining and knowledge discovery. The theory is considered to have good prospects for application to the development of intelligence systems. However, some problems with handling fuzzy data have been spotted in traditional RS, so extending the existing theories and methods of RS to deal with fuzzy data is becoming an important topic of research.

An appropriate reasoning schema plays an important role in improving the inference performance of an intelligence system. As an influential inference

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method in the approximate reasoning field, similarity-based approximate reasoning (SAR) has been applied successfully in some fields (Turksen and Zhao, 1988), but still needs to be improved in terms of its inference mechanism. Combining RS with the interval-valued fuzzy set (IVFS) (Turksen, 1986; Gorzalczany, 1987), we present an approach for extracting a knowledge model from a complex system. We then propose a novel inference method based on the similarity degree of IVFS. We deal with the fuzzy data based on IVFS because it is considered to be more flexible than a general fuzzy set.

2 Literature review

Zadeh (1965) proposed the theory of fuzzy sets (FS). For an arbitrary fuzzy set A, each object u in a universe of discourse U is assigned a real value between 0 and 1, called the membership function of A. However, due to the difficulties in obtaining the membership function of the fuzzy set in some applications, an extensional fuzzy set, named the interval-valued fuzzy set (IVFS), was proposed (Turksen, 1986; Gorzalczany, 1987). This allows the membership function to be represented by an interval-based value instead of a point-based value.

An IVFS A of a universe U can be represented by a set of ordered pairs $A=\{(x, [\mu_A'(x), \mu_A''(x)]): x\in U\}$, where (μ_A', μ_A'') is a couple of mappings from U to [0, 1] such that $0\leq\mu_A'(x)\leq\mu_A''(x)\leq 1$. The subinterval $[\mu_A'(x), \mu_A''(x)]$ indicates that the membership grade of x in A is between $\mu_A'(x)$ and $\mu_A''(x)$, and all the IVFSs of U are written as IVFS(U).

Let $A\in$IVFS(U). Then the lower bound, upper bound, kernel, and mid-value of A are represented, respectively, by

$$A' = \{(x, \mu_A'(x)) : x\in U\},\quad A'' = \{(x, \mu_A''(x)) : x\in U\},$$
$$K_A = \{(x, K_A(x)) : K_A(x) = \mu_A'(x) + \mu_A''(x), x\in U\},$$
$$\Phi_A = \{(x, \Phi_A(x)) : \Phi_A(x) = (\mu_A'(x) + \mu_A''(x))/2, x\in U\}.$$

Atanassov (1986) introduced the intuitionistic fuzzy set (IFS) through adding a non-membership function of [0, 1], and Gau and Buehrer (1993) presented the vague set (VS) by defining a true membership function, a false membership function, and a subinterval of [0, 1]. Since these extensional fuzzy sets have been proven to be essentially isomorphic (Bustince and Burillo, 1996; Deschrijver and Kerre, 2003), where there is no chance of confusion they are all referred to as IVFSs in this article. Compared with the traditional FS, IVFS has a larger information capacity and a more flexible representation form for representing fuzzy information. Therefore, it has begun to be used in fields like approximate reasoning, decision making (Wan, 2010), pattern recognition (Yang and Wang, 2010), fault diagnosis (Zheng *et al.*, 2008), and fuzzy control (Ou *et al.*, 2009).

It can be viewed as a generalization of fuzzy inference to apply IVFS in the field of approximate reasoning. Cornelis *et al.* (2004), Yang and Chen (2011), and Feng and Liu (2012) investigated a series of operators appropriate for the compositional rule of inference (CRI) method. Chen (1997), Fan *et al.* (2008), Hai and Lei (2010), and Zhang and Jiang (2010) discussed the representations for similarity measures of IVFS, and then presented the SAR method regarding unidirectional inference and bidirectional inference.

Rough set theory has been widely used in the fields of knowledge discovery, data mining, fault diagnosis, machine learning, expert systems, and decision support since it was proposed by Pawlak (1982). However, in the real world the data in information systems is often fuzzy, such as ‘high’ and ‘low’ temperatures or ‘fast’ and ‘slow’ speeds. To enable RS to deal with fuzzy data more efficiently, Pawlak (1982) pointed out that RS and FS are complementary rather than exclusive. Dubois and Prade (1990) considered that RS and FS are two mathematical methods for handling two different uncertainties, including roughness and fuzziness, and introduced the notions of fuzzy rough set and rough fuzzy set. Guan and Wang (2006) discussed the problems regarding attribute reduction and rule extraction in set-valued information systems based on maximal tolerant classes. Jensen and Shen (2009) presented an approach for feature selection based on fuzzy-rough sets. Cornelis and Jensen (2010) addressed two fuzzy decision reduction methods in continuous-valued attribute information systems. In view of the advantages of IVFS in representing fuzzy information, some researchers devoted themselves to combining IVFS with RS. Gong *et al.* (2008) investigated RS theory for interval-valued fuzzy information systems, and presented an approach to attribute reduction and rule acquisition. Cornelis *et al*. (2003), Qiu (2006), and Liang (2007) addressed three integration models of IVFS and RS: a generalized interval-valued fuzzy rough set, a rough IVFS, and an interval-valued fuzzy rough set based on t-norm. Feng and Wang (2010) provided a definition of a rough interval-valued set in crisp approximate space, and then put forward a method of knowledge extraction for an interval-valued fuzzy objective information system.

3 Modeling based on interval-valued fuzzy rough set

Combining RS and IVFS, in this section we present a modeling method based on interval-valued fuzzy rough set. The method is applicable to extracting a fuzzy knowledge model represented by "If-Then" form from empirical data running in a system. The main procedures of the proposed method are as follows: (1) obtaining raw data, (2) pretreatment of data, (3) interval-valued fuzzification of data, (4) attribute reduction, and (5) rule extraction. In (3), through defining linguistic values and an interval-valued membership function, an information system containing continuous-valued data can be translated into an interval-valued fuzzy information system containing interval-valued fuzzy data. Attribute reduction is the kernel step of the proposed modeling method; it includes subprocedures like establishment of an interval-valued fuzzy rough set, definition of an interval-valued fuzzy positive region, and reduction algorithm implementation.

3.1 Interval-valued fuzzy information system

An interval-valued fuzzy information system is defined by a pair (U, A), which can be formally represented as an interval-valued fuzzy information table. U is a nonempty universe of discourse. A is a limited attribute set, in which a concept family composed of several fuzzy concepts can be induced by every attribute $a\in A$. Each concept is described by an interval-valued fuzzy subset of the universe U, i.e.,

$$\mathbb{C}_a=\{F_a^i\in \text{IVFS}(U): i=1,2,\cdots,p\}.$$

In an interval-valued fuzzy information system (U, A), if $A=C\cup\{d\}$, $C\cap\{d\}=\varnothing$, where C and d are a conditional attribute set and a decision attribute, respectively, then (U, $C\cup\{d\}$) is called an interval-valued fuzzy decision system. Such a system can be formally expressed by a two-dimensional table named the interval-valued fuzzy decision table. An interval-valued fuzzy decision table of automobile prices is shown in Table 1. The membership function of interval-valued fuzzy subsets is often determined by the experience of domain experts.

3.2 Interval-valued fuzzy rough approximation

Let R be an equivalence relation of U, and $P(U)$ be the power set of U. According to RS theory, a crisp equivalent class family $\mathbb{C}=\{X_i\in P(U): i=1, 2, \dots, m\}$ can be derived from the crisp partition U/R, and the pair (U, $\mathbb{C}$) is called a crisp approximation space. Since each X_i is describable by a crisp concept, U/R is actually a concept family formed by several concepts. Let $Y\in P(U)$. The lower and upper rough

Table 1 Interval-valued fuzzy decision table of automobile prices

Attribute		x_1	x_2	x_3	x_4	x_5	x_6
Performance	Good	[0.2, 0.4]	[0.3, 0.4]	[0.7, 0.9]	[0.8, 0.8]	[0.2, 0.3]	[0.1, 0.4]
	Poor	[0.8, 0.9]	[0.7, 0.9]	[0.2, 0.3]	[0.4, 0.5]	[0.7, 0.8]	[0.7, 0.7]
Fuel consumption	High	[0.1, 0.3]	[0.1, 0.2]	[0.2, 0.3]	[0.3, 0.5]	[0.1, 0.3]	[0.6, 0.7]
	Medium	[0.4, 0.5]	[0.3, 0.5]	[0.3, 0.5]	[0.6, 0.8]	[0.2, 0.5]	[0.4, 0.5]
	Low	[0.7, 0.8]	[0.7, 0.9]	[0.8, 0.8]	[0.4, 0.4]	[0.7, 0.8]	[0.3, 0.4]
Maximum distance	Long	[0.2, 0.4]	[0.2, 0.3]	[0.4, 0.4]	[0.2, 0.4]	[0.2, 0.3]	[0.7, 0.8]
	Medium	[0.3, 0.4]	[0.2, 0.4]	[0.7, 0.8]	[0.3, 0.4]	[0.3, 0.4]	[0.3, 0.4]
	Short	[0.7, 0.8]	[0.7, 0.8]	[0.4, 0.5]	[0.6, 0.8]	[0.7, 0.8]	[0.1, 0.1]
Price	High	[0.2, 0.3]	[0.1, 0.3]	[0.1, 0.2]	[0.3, 0.5]	[0.2, 0.3]	[0.7, 0.8]
	Medium	[0.2, 0.4]	[0.3, 0.4]	[0.2, 0.3]	[0.8, 0.9]	[0.3, 0.3]	[0.3, 0.4]
	Low	[0.8, 0.8]	[0.6, 0.8]	[0.8, 0.9]	[0.3, 0.4]	[0.7, 0.9]	[0.1, 0.2]

approximations of Y in $(U, \mathbb{C})$ are represented, respectively, by

$$\underline{\mathbb{C}}(Y) = \bigcup_{X_i \subseteq Y} X_i, \quad \overline{\mathbb{C}}(Y) = \bigcup_{X_i \cap Y \neq \varnothing} X_i.$$

The pair $(\underline{\mathbb{C}}(Y), \overline{\mathbb{C}}(Y))$ is called the rough set in $(U, \mathbb{C})$, and Y is definable when $\underline{\mathbb{C}}(Y) = \overline{\mathbb{C}}(Y)$; otherwise, Y is rough definable in $(U, \mathbb{C})$.

Through extending crisp partition to weak fuzzy partition, Kuncheva (1992) presented a fuzzy rough set based on inclusion measures of fuzzy sets, in which the positive, negative, and boundary regions of a fuzzy subset $Y \in \mathrm{FS}(U)$ with respect to the weak fuzzy partition $\mathbb{C}$ are expressed, respectively by

$$\mathrm{POS}_{\mathbb{C}}^{\lambda_1}(Y) = \bigcup \{F_i \in \mathbb{C}\colon D(F_i, Y) \geq \lambda_1\},$$
$$\mathrm{NEG}_{\mathbb{C}}^{\lambda_2}(Y) = \bigcup \{F_i \in \mathbb{C}\colon D(F_i, Y) \leq \lambda_2\},$$
$$\mathrm{BND}_{\mathbb{C}}^{\lambda_2, \lambda_1}(Y) = \bigcup \{F_i \in \mathbb{C}\colon \lambda_2 < D(F_i, Y) < \lambda_1\},$$

where $\mathbb{C} = \{F_i \in \mathrm{FS}(U) : i = 1, 2, \ldots, m\}$ is a concept family composed of several fuzzy concepts, and $D(F_i, Y)$ is the inclusion degree of F_i in Y.

Now, consider generalizing Kuncheva's model in an interval-valued fuzziness environment. Assume that $\mathbb{C} = \{F_i \in \mathrm{IVFS}(U) : i = 1, 2, \cdots, m\}$ is an interval-valued fuzzy collection, and each interval-valued fuzzy subset F_i corresponds to a fuzzy concept. $\mathbb{C}$ is called a weak interval-valued fuzzy partition of U if, and only if, $\forall F_i, F_j \in \mathbb{C}$, $i \neq j$,

(P1) $\inf_x \max K_{F_i}(x) > 0$,
(P2) $\sup_x \max \{K_{F_i}(x), K_{F_j}(x)\} < 2$,

where $K_{F_i} \in \mathrm{FS}(U)$ is the kernel of F_i. P1 suggests that the universe U should be fully covered by the concept family $\mathbb{C}$, and P2 implies that in the universe U no object belongs fully to two different fuzzy concepts. For simplicity, in the following parts a weak interval-valued fuzzy partition is named a weak partition.

Let $\mathbb{C}$ be a weak partition of U. An arbitrary interval-valued fuzzy subset in $\mathbb{C}$ is called an interval-valued fuzzy class, and a pair $(U, \mathbb{C})$ is called an interval-valued fuzzy approximation space. Given two real numbers α and β such that $0 \leq \alpha < \beta \leq 1$, the β-lower and α-upper interval-valued fuzzy rough approximations of an interval-valued fuzzy subset $Y \in \mathrm{IVFS}(U)$ in $(U, \mathbb{C})$ can be represented, respectively, by

$$\underline{\mathbb{C}}_\beta(Y) = \bigcup \{F_i \in \mathbb{C}\colon D(F_i, Y) \geq \beta\},$$
$$\overline{\mathbb{C}}^\alpha(Y) = \bigcup \{F_i \in \mathbb{C}\colon D(F_i, Y) \geq \alpha\},$$

where $D(F_i, Y)$ denotes the inclusion degree of F_i in Y. The pair $(\underline{\mathbb{C}}_\beta(Y), \overline{\mathbb{C}}^\alpha(Y))$ is called an interval-valued fuzzy rough set in $(U, \mathbb{C})$, and the β-positive, α-negative, and (α, β)-boundary regions of Y in $(U, \mathbb{C})$ are represented, respectively, by

$$\mathrm{POS}_{\mathbb{C}}^{\beta}(Y) = \bigcup \{F_i \in \mathbb{C}\colon D(F_i, Y) \geq \beta\},$$
$$\mathrm{NEG}_{\mathbb{C}}^{\alpha}(Y) = \bigcup \{F_i \in \mathbb{C}\colon D(F_i, Y) \leq \alpha\},$$
$$\mathrm{BND}_{\mathbb{C}}^{\alpha, \beta}(Y) = \bigcup \{F_i \in \mathbb{C}\colon \alpha < D(F_i, Y) < \beta\}.$$

Thus, an interval-valued fuzzy rough set is largely determined by the inclusion measures of IVFS. The inclusion function $\mathrm{IVFS}(U) \times \mathrm{IVFS}(U) \to [0, 1]$ should satisfy the following conditions for all $A, B \in \mathrm{IVFS}(U)$:

(D1) If $A \subseteq B$ then $D(A, B) = 1$;
(D2) If $A \cap B = \varnothing$ then $D(A, B) = 0$.

Hence, the inclusion grade of A in B can be represented as follows:

$$D(A, B) = \frac{\mathrm{card}(K_{AB}^1)}{\mathrm{card}(K_A^1)}, \tag{1}$$

where card(·) is the cardinal number of a crisp set, and

$$K_A^1 = \{x \in U : K_A(x) > 1\},$$
$$K_{AB}^1 = \{x \in U : K_A(x) > 1, K_B(x) > 1\}.$$

Take $D(A, B) = 1$ when $\mathrm{card}(K_A{}^1) = 0$.

3.3 Interval-valued fuzzy positive region

For an interval-valued fuzzy information system (U, A), each attribute induces a fuzzy concept family, and therefore a new concept family can be derived from multiple attributes in (U, A). Let $B\subseteq A$, $\mathbb{C}_a$ be a weak partition derived from $a\in B$, and $F_a\in\text{IVFS}(U)$ be an arbitrary interval-valued fuzzy class in $\mathbb{C}_a$. All concepts induced by B form a weak partition $\mathbb{C}_B$ such that

$$\mathbb{C}_B = \underset{a\in B}{\otimes}(\mathbb{C}_a) = \{F_B \in \text{IVFS}(U): F_B = \bigcap_{a\in B} F_a\},$$

where $\otimes$ is the Cartesian product of collections. For example, let $\{b, c\}\subseteq A$. Then a weak partition derived from $\{b, c\}$ is represented as

$$\mathbb{C}_{\{b,c\}} = \mathbb{C}_b \otimes \mathbb{C}_c = \{F_{\{b,c\}} \in \text{IVFS}(U): F_{\{b,c\}} = F_b \cap F_c\}.$$

Here, F_b and F_c are the interval-valued fuzzy classes in $\mathbb{C}_b$ and $\mathbb{C}_c$, respectively. Because a weak partition corresponds one-to-one to an attribute in (U, A), the attribute can be viewed as a label of knowledge represented by weak partition, named the labeled attribute. This implies that an attribute and a weak partition are interchangeable in an interval-valued fuzzy information system.

Let $B\subseteq A$ and $Y\in\text{IVFS}(U)$. Then the β-lower and α-upper interval-valued fuzzy rough approximations of Y with respect to B are defined, respectively, by

$$\underline{B}_\beta(Y) = \bigcup\{F_B \in \mathbb{C}_B:\ D(F_B, Y) \geq \beta\},$$
$$\overline{B}^\alpha(Y) = \bigcup\{F_B \in \mathbb{C}_B:\ D(F_B, Y) \geq \alpha\}.$$

Thus, the β-positive region of Y with respect to B is represented by

$$\text{POS}_B^\beta(Y) = \underline{B}_\beta(Y).$$

Suppose $(U, C\cup\{d\})$ is an interval-valued fuzzy decision system and $B\subseteq C$. The β-positive region of d with respect to B can be denoted by

$$\text{POS}_B^\beta(d) = \bigcup_{F_d\in\mathbb{C}_d}\{F_B \in \mathbb{C}_B: D(F_B, F_d) \geq \beta\}, \quad (2)$$

where $\mathbb{C}_B$ and $\mathbb{C}_d$ are the weak partitions induced by conditional attribute subset B and decision attribute d, respectively, and F_B and F_d are the interval-valued fuzzy classes in $\mathbb{C}_B$ and $\mathbb{C}_d$, named the conditional class and decision class, respectively. Denote $\text{POS}_B^\beta(d)$ as POS_B^β for simplicity.

3.4 Attribute reduction in an interval-valued fuzzy information system

In an interval-valued fuzzy decision system $(U, C\cup\{d\})$, let $P(C)$ be the power set of conditional attribute set C. Given an increasing measure function M^β: $P(C)\rightarrow[0, 1]$ such that $M^\beta(C)=1$. If $M^\beta(B)=1$ for every $B\in P(C)$, and $M^\beta(B-\{a\})\leq 1$ for every $a\in B$, then B is a β-reduction of C with respect to d.

Generally, in a decision system the dependency of the decision attribute with respect to the conditional attribute is applicable for the measures of attribute reduction. The target of reduction is to find a minimal attribute set with the same dependency degree as the conditional attribute set. To extend the dependency measures in traditional RS, a normalized function γ^β: $P(C)\rightarrow[0, 1]$ can be defined by

$$\gamma^\beta(B) = \min\left\{\sum_x \Phi_{\text{POS}_B^\beta}(x) \Big/ \sum_x \Phi_{\text{POS}_C^\beta}(x), 1\right\} \quad (3)$$

for every $B\in P(C)$, where Φ_F is the mid-value of an IVFS F.

There are several approaches to implementing reduction in a crisp information system. The QuickReduct algorithm is an effective method to compute the minimal reduction without exhaustively generating all possible attribute subsets (Shen and Chouchoulas, 2000). It begins with an empty set and adds in turn, one at a time, those attributes that cause the greatest increase in dependency degree, until this produces its maximum possible value for the information system (usually 1). Apparently, the change of dependency is used as the heuristic information in this algorithm; i.e., the attributes that result in a rapid increase of dependency measures should be included in the minimal reduction.

Based on this algorithm, an interval-valued fuzzy rough based QuickReduct algorithm applicable for an interval-valued fuzzy decision system is given in Algorithm 1.

Algorithm 1 Interval-valued fuzzy rough based QuickReduct algorithm

RED←{}
do
  T←RED
  $M^{\beta}_{\text{best}}=0$
  for every $a\in(C-\text{RED})$
    if $M^{\beta}_{\text{RED}\cup\{a\}}>M^{\beta}_{\text{best}}$
      T←RED∪{a}
      $M^{\beta}_{\text{best}}\leftarrow M^{\beta}_{\text{RED}\cup\{a\}}$
    end if
  end for
  RED←T
until $M^{\beta}_{\text{best}}=1$
return RED

Example 1 Compute the attribute reduction in Table 1, provided that β=0.6.

Denote ‘performance’, ‘fuel consumption’, and ‘maximum distance’ as a, b, and c, respectively. According to Eq. (2), we have

$$\begin{aligned}\text{POS}^{0.6}_{\{a,b,c\}}&=\{[0.7,\ 0.8],\ [0.7,\ 0.8],\ [0.7,\ 0.8],\\&\quad[0.6,\ 0.8],\ [0.7,\ 0.8],\ [0.6,\ 0.7]\},\\\text{POS}^{0.6}_{\{a\}}&=\{[0.8,\ 0.9],\ [0.7,\ 0.9],\ [0.2,\ 0.3],\\&\quad[0.4,\ 0.5],\ [0.7,\ 0.8],\ [0.7,\ 0.7]\},\\\text{POS}^{0.6}_{\{b\}}&=\{[0.7,\ 0.8],\ [0.7,\ 0.9],\ [0.8,\ 0.8],\\&\quad[0.6,\ 0.8],\ [0.7,\ 0.8],\ [0.6,\ 0.7]\},\\\text{POS}^{0.6}_{\{c\}}&=\{[0.7,\ 0.8],\ [0.7,\ 0.8],\ [0.7,\ 0.8],\\&\quad[0.6,\ 0.8],\ [0.7,\ 0.8],\ [0.3,\ 0.4]\}.\end{aligned}$$

By Eq. (3), we obtain

$$\gamma^{0.6}_{\{a\}}=0.905,\quad \gamma^{0.6}_{\{b\}}=1,\quad \gamma^{0.6}_{\{c\}}=0.964.$$

Thus, RED={b} is a reduction of C with respect to d.

3.5 Rule extraction

Let RED⊂C be a reduction in an interval-valued fuzzy decision system (U, C∪{d}), and $\mathbb{C}_{\text{RED}}$ and $\mathbb{C}_d$ be the weak partitions derived from RED and d, respectively. Applying Eq. (1), we may calculate the inclusion degree $D(F_{\text{RED}}, F_d)$ of F_{RED} in F_d. Here, $F_{\text{RED}}=\bigcap_{a\in\text{RED}}F_a$ is an interval-valued fuzzy conditional class in $\mathbb{C}_{\text{RED}}$, and F_d is an interval-valued fuzzy decision class in $\mathbb{C}_d$.

Given a real number $\lambda\in[0, 1]$, if

$$\begin{cases}D(F_{\text{RED}},F_d)\geq\lambda,\\ \sup K_{F_{\text{RED}}}(x)>1,\end{cases}\tag{4}$$

for every $F_{\text{RED}}\in\mathbb{C}_{\text{RED}}$, a fuzzy decision rule set extracted from (U, C∪{d}) can be represented by

$$\underset{a\in\text{RED}}{\wedge}(a=\text{Des}(F_a))\rightarrow(d=\text{Des}(F_d)),$$

where Des(·) denotes the semantic description of an IVFS (i.e., linguistic value), and the inclusion measures may be interpreted as confidence degrees of a rule.

Example 2 Extract decision rules according to the reduction results of Example 1, provided that λ=0.8.

For RED={b}, calculate the inclusion grade between the conditional classes in $\mathbb{C}_b$ and the decision classes in $\mathbb{C}_d$, applying Eq. (1). The computation results are shown in Table 2.

Table 2 Inclusion degree computation

	$D(F_b^{(j)},F_d^{(k)})$		
	$F_b^{(1)}$	$F_b^{(2)}$	$F_b^{(3)}$
$F_d^{(1)}$	1	0	0
$F_d^{(2)}$	0	1	0
$F_d^{(3)}$	0	0	1

j, k=1, 2, 3

According to the conditions given in Eq. (4), we obtain a decision rule set as follows:

$$\begin{aligned}&\text{(R1)}\ \ b=\text{Des}(F_b^{(1)})\rightarrow d=\text{Des}(F_d^{(1)}),\\&\text{(R2)}\ \ b=\text{Des}(F_b^{(2)})\rightarrow d=\text{Des}(F_d^{(2)}),\\&\text{(R3)}\ \ b=\text{Des}(F_b^{(3)})\rightarrow d=\text{Des}(F_d^{(3)}).\end{aligned}$$

4 Model inference with the SAR method

Applying the interval-valued fuzzy rough set theory, we may obtain a fuzzy knowledge model from the data set running in an information system. The following task is to employ the fuzzy rules acquired to deduce results by selecting an appropriate reasoning strategy.

4.1 Similarity-based approximate reasoning

As the theoretical foundation of fuzzy control, fuzzy inference has achieved successful applications in many fields. The basic fuzzy modus ponens (FMP) that is often studied can be represented as follows:

Rule	If	x is A	Then	y is B
Case		x is A^*		
Conclusion				y is B^*

where x and y are linguistic variables, A and A^* are fuzzy subsets of universe X, and B and B^* are fuzzy subsets of universe Y. Zadeh (1965) proposed the well-known compositional rule of inference (CRI), where a fuzzy relation R between the antecedent part A and the consequent part B is constructed, and then the conclusion B^* can be derived from the compositional operation of A^* and R. Many fuzzy systems are based on Zadeh's CRI. Although the CRI algorithm had achieved notable success in various fields such as fuzzy control, expert systems, and decision-making support, some defects of this method were found in terms of the inference mechanism. This led to the development of another important approximation inference approach—similarity-based approximate reasoning (SAR) (Turksen and Zhao, 1988; Yeung and Tsang, 1997; Raha, 2008; Wang *et al.*, 2008). The SAR approach is also suitable for an interval-valued fuzzy environment, as mentioned in Section 2.

In existing SAR methodology, in a rule-based system reasoning is based on the computation of a similarity grade $S(A^*, A)$ between the fact A^* and the antecedent part A. The inference result B^* is obtained by directly modifying the consequent part B with $S(A^*, A)$, i.e., $B^*=f(S(A^*, A), B)$, where f is a modification function. Evidently, the inherent relation between the antecedent and the consequent is largely ignored, and the same result will be obtained using the SAR method when A^* and A are interchanged. Thus, this result seems somewhat unconvincing because the inference is not always influenced by every change in the input fact and the antecedent part.

The conventional CRI does not consider similarity measures in the process of inference. The existing SAR methods modify directly the consequent part of a rule based on the similarity measure, and therefore the conclusion obtained is independent of a conditional statement. Based on the IVFS, we attempt below to integrate the above techniques to produce an adequate theory of similarity-based approximate reasoning.

4.2 Similarity measures of interval-valued fuzzy sets

Chen (1994) presented a representation for similarity degree based on fuzzy vectors. Let $A, B\in$ FS(X). Then the similarity grade $S(A, B)$ between A and B can be represented by

$$S(A,B)=\frac{\sum_x \mu_A(x)\cdot\mu_B(x)}{\max\left\{\sum_x \mu_A^2(x), \sum_x \mu_B^2(x)\right\}}. \tag{5}$$

Here, the sum-product operation represents the product of fuzzy vectors A and B.

Some factors should be considered to provide a representation for similarity measure $\tilde{S}(A,B)$ of IVFS. Given a mapping $\tilde{S}:\text{IVFS}(X)\times\text{IVFS}(X)\rightarrow$ [0, 1] such that

(S1) $\tilde{S}(A,B)=\tilde{S}(B,A)$,

(S2) if $A=B$ then $\tilde{S}(A,B)=1$,

(S3) if $A\cap B=\varnothing$ then $\tilde{S}(A,B)=0$, and

(S4) if $A\subseteq B\subseteq C$ then $\tilde{S}(A,C)<\min\{\tilde{S}(A,B), \tilde{S}(B,C)\}$,

for every A, B, and $C\in$IVFS(X), then $\tilde{S}$ is called the similarity function of IVFS.

From the definition given above, a similarity measure $\tilde{S}(A,B)$ between A and B is represented as

$$\tilde{S}(A,B)=[S(A',B')+S(A'',B'')+2S(K_A,K_B)]/4, \tag{6}$$

where $S(\cdot,\cdot)$ are the similarity grades derived from Eq. (5). F', F'', and K_F are the upper bound, lower bound, and kernel, respectively, of an IVFS $F\in$ IVFS(X).

It is easy to verify that this similarity measure satisfies conditions S1–S4. Thus, the similarity grades of the subscript, superscript, and kernel of IVFS are incorporated into Eq. (6), where the weighted coefficients are 0.25, 0.25, and 0.5, respectively.

4.3 An improved SAR method

According to the principles of CRI, a conditional statement (rule) "If x is A then y is B" can be translated into an interval-valued fuzzy relation of $X\times Y$, written as $R(A, B)$. To construct the relation, some potential operators can be used. For example, the relation $R(A, B)$ formed by the extensional KD-implication can be represented as

$$\mu'_{R(A,B)}(x,y)=\min\{1-\mu''_A(x),\mu'_B(y)\},$$
$$\mu''_{R(A,B)}(x,y)=\min\{1-\mu'_A(x),\mu''_B(y)\}.$$

Given an input variable A^*, there should be an interval-valued fuzzy relation $R(A^*, B)$ such that

$$R(A^*,B)=A^*\cap R(A,B).$$

Then B^* can be deduced by supremum projection of $R(A^*, B)$, i.e.,

$$\begin{aligned}\mu'_{B^*}(y)&=\sup_x \mu'_{R(A^*,B)}(x,y)\\&=\sup_x \min\{\mu'_{A^*}(x),\mu'_{R(A,B)}(x,y)\},\\\mu''_{B^*}(y)&=\sup_x \mu''_{R(A^*,B)}(x,y)\\&=\sup_x \min\{\mu''_{A^*}(x),\mu''_{R(A,B)}(x,y)\}.\end{aligned}$$

The CRI method fails to incorporate matching computation into its inference procedures and therefore, in some application occasions the deduced results are not always satisfactory in terms of inference precision.

The mechanism of SAR is to conclude a result by modifying the consequent part of a rule with a similarity measure. According to this principle, in a rule-based system we may first calculate the similarity grade $\tilde{S}(A^*,A)$ between a fact A^* and antecedent part A. Next, an interval-valued fuzzy relation $R(A^*, B)$ between A^* and B, named the induced relation, is obtained by modifying the relation $R(A, B)$ with $\tilde{S}(A^*,A)$. Finally, the result B^* can be deduced by the projection operation on the induced relation $R(A^*, B)$.

Given a conditional statement "If x is A then y is B", the following cases should be taken into account to acquire an induced relation using the similarity measure:

Case 1: If A^* is equal to A, then $R(A^*, B)$ is equal to $R(A, B)$. This means we should not make any modification of $R(A, B)$ when $\tilde{S}(A^*,A)=1$.

Case 2: If A^* is completely dissimilar to A, then nothing can be concluded from the given conditional statement, i.e., B^* is equal to empty. Since

$$\mu'_{B^*}(y)=\sup_x \mu'_{R(A^*,B)}(x,y),$$
$$\mu''_{B^*}(y)=\sup_x \mu''_{R(A^*,B)}(x,y),$$

for every $(x, y)\in X\times Y$,

$$\mu'_{R(A^*,B)}(x,y)=\mu''_{R(A^*,B)}(x,y)=0.$$

That is, $R(A^*, B)=\varnothing$ when $\tilde{S}(A^*,A)=0$.

Case 3: As $\tilde{S}(A^*,A)$ changes from 0 to 1, $R(A^*, B)$ changes from $\varnothing$ to $R(A, B)$. This means $R(A^*, B)$ is transformed from an unknown state into a specific state.

From the cases mentioned above, a quantitative relationship between the induced relation and similarity measure can be deduced below. Let

$$\tilde{S}(A^*,A)=s,\ \mu'_{R(A,B)}(x,y)=r',\ \mu''_{R(A,B)}(x,y)=r'',$$
$$\mu'_{R(A^*,B)}(x,y)=r^{*\prime},\ \mu''_{R(A^*,B)}(x,y)=r^{*\prime\prime}.$$

As in cases 1 and 2, we have

$$r^{*\prime}=\begin{cases}0, & s=0,\\ r', & s=1,\end{cases}\quad r^{*\prime\prime}=\begin{cases}0, & s=0,\\ r'', & s=1,\end{cases}$$

and by case 3, we obtain

$$r^{*\prime}=r^{*\prime}(s)=T(s,r'),\quad r^{*\prime\prime}=r^{*\prime\prime}(s)=T(s,r''),$$

where T is a continuous t-norm. Thus, a modification schema for generating the induced relation $R(A^*, B)$ may be represented by

$$\begin{cases}\mu'_{R(A^*,B)}(x,y)=T(s,\mu'_{R(A,B)}(x,y)),\\ \mu''_{R(A^*,B)}(x,y)=T(s,\mu''_{R(A,B)}(x,y)),\end{cases}\tag{7}$$

and the inference result B^* is obtained by the supremum projection over $R(A^*, B)$, i.e.,

$$\begin{cases}\mu'_{B^*}(y)=\sup_x T(s,\mu'_{R(A,B)}(x,y)),\\ \mu''_{B^*}(y)=\sup_x T(s,\mu''_{R(A,B)}(x,y)).\end{cases} \tag{8}$$

Apparently, there are some differences in inference mechanism between the conventional CRI and the proposed method. A logical interpretation for the CRI method is: from "x is A^* and (x, y) is $R(A, B)$" infer "y is B^*". For the proposed method, the inference mechanism can be interpreted as: from "A^* is similar to A and (x, y) is $R(A, B)$" infer "y is B^*", where the connective 'and' is associated with t-norm operation. The main procedures of our proposed algorithm are summarized below:

Step 1 (translation): Translate a conditional statement into an interval-valued fuzzy relation $R(A, B)$ by some possible operators.

Step 2 (matching): Compute $\tilde{S}(A^*, A)=1$ applying a suitable similarity measure.

Step 3 (modification): Obtain an induced relation $R(A^*, B)$ by a modification schema.

Step 4 (projection): Deduce B^* by supremum projection over $R(A^*, B)$.

In step 1, we have to select a suitable operator I to translate a rule. There are about 18 operators applicable to construct $R(A, B)$ in the CRI method, which are often classified into two main classes. The first class is called the extensional 'and' operators, such as the Mamdani operator, the Larson operators, and the bounded product. The second class is called the extensional 'implication' operators, including the well-known S-implicator and the R-implicator. The following proposition will provide a clue for how to select an operator I for the construction of $R(A, B)$:

Proposition 1 Suppose A is normal and does not completely cover the domain, and s=1.

(1) If I is both left and right increasing, $B^*=B$;

(2) If I is left decreasing and right increasing, $B^*=Y$.

Proof

(1) Since I is both increasing, then

$$\mu'_{R(A,B)}(x,y)=I(\mu'_A(x),\mu'_B(y)).$$

By Eq. (8),

$$\begin{aligned}\mu'_{B^*}(y)&=\sup_x T(s,I(\mu'_A(x),\mu'_B(y)))\\ &=T(s,\sup_x I(\mu'_A(x),\mu'_B(y)))\\ &=T(s,I(\sup_x \mu'_A(x),\mu'_B(y))).\end{aligned}$$

Again, A is normal. Then

$$\mu'_{B^*}(y)=T(s,I(1,\mu'_B(y)))=T(s,\mu'_B(y)).$$

Similarly, we have

$$\mu''_{B^*}(y)=T(s,\mu''_B(y)).$$

Hence, we obtain $B^*=B$ by s=1.

(2) Since I is left decreasing and right increasing, we have

$$\mu'_{R(A,B)}(x,y)=I(\mu''_A(x),\mu'_B(y)).$$

By Eq. (8),

$$\begin{aligned}\mu'_{B^*}(y)&=\sup_x T(s,I(\mu''_A(x),\mu'_B(y)))\\ &=T(s,\sup_x I(\mu''_A(x),\mu'_B(y)))\\ &=T(s,I(\inf_x \mu''_A(x),\mu'_B(y))).\end{aligned}$$

Again, A does not completely cover the domain, i.e.,

$$\inf_x \mu'_A(x)=\inf_x \mu''_A(x)=0.$$

Then

$$\mu'_{B^*}(y)=T(s,I(0,\mu'_B(y)))=T(s,1)=s.$$

Similarly, we may deduce $\mu''_{B^*}(y)=s$. Hence, we obtain $B^*=Y$ by s=1.

Remark 1 If an operator selected for constructing an interval-valued fuzzy relation is both increasing, then the inference result satisfies the reductive property when the antecedent part is normal. Furthermore, because

$$\mu'_{B^*}(y)=T(s,\mu'_B(y)),\ \mu''_{B^*}(y)=T(s,\mu''_B(y)),$$

we obtain

$$\mu'_{B^*}(y)=s\cdot\mu'_B(y),\ \mu''_{B^*}(y)=s\cdot\mu''_B(y), \tag{9}$$

when t-norm takes the algebraic product. Eq. (9) is exactly the extension of Turksen's reduction type (Turksen and Zhao, 1988). For an arbitrary antecedent part $A\in$IVFS(X), we obtain

$$\begin{cases}\mu'_{B^*}(y)=T(s,I(\sup_x \mu'_A(x),\mu'_B(y))),\\ \mu''_{B^*}(y)=T(s,I(\sup_x \mu''_A(x),\mu''_B(y))).\end{cases} \tag{10}$$

Eq. (10) can be seen as a general representation of SAR proposed in this section, while Turksen's reduction form is a special case of Eq. (10).

Remark 2 If an operator selected for constructing an interval-valued fuzzy relation is hybrid monotonic, then the inference result equals the whole set as the antecedent part does not completely cover the domain. In this case, the result becomes the most unspecific case because $B^*=Y$ means "B^* is anything" from the viewpoint of semantics.

Proposition 2 Suppose $A\in$IVFS(X) is normal. Then, $B^*\subseteq B$.

Proof Since A is normal, by Eq. (10), we have

$$\begin{aligned}\mu'_{B^*}(y)&=T(s,I(\sup_x \mu'_A(x),\mu'_B(y)))\\&=T(s,I(1,\mu'_B(y)))\le I(1,\mu'_B(y))=\mu'_B(y),\\ \mu''_{B^*}(y)&=T(s,I(\sup_x \mu''_A(x),\mu''_B(y)))\\&=T(s,I(1,\mu''_B(y)))\le I(1,\mu''_B(y))=\mu''_B(y),\end{aligned}$$

for every $y\in Y$. Hence, we obtain $B^*\subseteq B$.

Remark 3 If the antecedent part of a rule is normal, then the conclusion B^* is reducing with respect to B, and B^* is reducible when $A^*=A$.

Example 3 Suppose that the antecedent part, the consequent part of a rule, and a case input are

$$\begin{aligned}&A=\{(x_1,[0.7,0.8]),(x_2,[0.4,0.5]),(x_3,[0.2,0.3]),(x_4,0)\},\\&B=\{(y_1,1),(y_2,[0.5,0.6]),(y_3,0)\},\\&A^*=\{(x_1,1),(x_2,[0.7,0.8]),(x_3,[0.3,0.3]),(x_4,0)\}.\end{aligned}$$

Compute B^* using Turksen's reduction type and the proposed SAR algorithm.

Applying Eq. (6), we have $\tilde{S}(A^*,A)=0.7$. According to the extensional Turksen reduction form (i.e., Eq. (9)), we have

$$B_1^*=\tilde{S}(A^*,A)\cdot B=\{(y_1,0.7),(y_2,[0.35,0.42]),(y_3,0)\}.$$

According to the algorithm proposed in this section, we first compute an interval-valued fuzzy relation of A and B using the Mamdani operator, i.e.,

$$R(A,B)=\begin{bmatrix}[0.7,0.8] & [0.5,0.6] & 0\\ [0.4,0.5] & [0.4,0.5] & 0\\ [0.2,0.3] & [0.2,0.3] & 0\\ 0 & 0 & 0\end{bmatrix},$$

and then we have an induced relation $R(A^*, B)$ by Eq. (7), i.e.,

$$\begin{aligned}R(A^*,B)&=\tilde{S}(A^*,A)\cdot R(A,B)\\&=\begin{bmatrix}[0.49,0.56] & [0.35,0.42] & 0\\ [0.28,0.35] & [0.28,0.35] & 0\\ [0.14,0.21] & [0.14,0.21] & 0\\ 0 & 0 & 0\end{bmatrix}.\end{aligned}$$

Here, the algebraic product is selected as a t-norm. Finally, B^* is obtained by a projection operation on $R(A^*, B)$, i.e.,

$$\begin{aligned}B_2^*&=\sup_x R(A^*,B)\\&=\{(y_1,[0.49,0.56]),(y_2,[0.35,0.42]),(y_3,0)\}.\end{aligned}$$

Assume that the fact A^* and the antecedent part A are interchanged. An identical result $B_1^{**}=B_1^*$ can be deduced from Turksen's method, whereas a different result

$$B_2^{**}=\{(y_1,[0.7,0.7]),(y_2,[0.35,0.42]),(y_3,0)\}\neq B_2^*$$

can be derived from our proposed SAR method.

Remark 4 From Example 3 we can see that every change in the FMP, as it appears in a premise or in a fact, may be incorporated into an induced interval-valued fuzzy relation. Furthermore, by projection operation on the induced relation, the inference result is influenced by changes in a fact or in the antecedent part of a rule.

5 Application to welding distortion prediction

Welding distortion of a ship's structure has a great impact on the quality, cost, and time involved in shipbuilding. Therefore, it is crucial to implement welding deformation prediction for ship structures. In this section, we attempt to investigate the regular pattern of distortion in the welding process, by means of the approach of interval-valued fuzzy rough set based knowledge modeling. Moreover, based on the fuzzy knowledge model acquired, similarity-based inference can be used to make a rapid prediction of the welding distortion of a marine structure, through which the technological means of welding deformation control is also presented. The basic procedures of modeling and inference are given in Fig. 1.

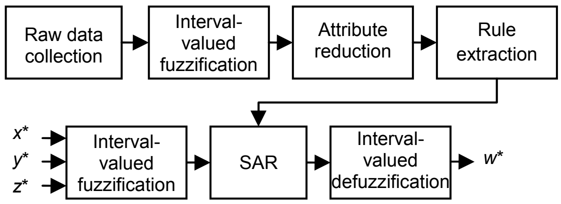

Fig. 1 Primary procedures of welding distortion prediction for marine structures

5.1 Acquisition for modeling data and fuzzification

To obtain raw data for modeling, a typical marine structural unit including high tensile steel plates, reinforcing ribs, and fixed irons was chosen for a welding experiment, where the sizes of the plate and rib were 1400 mm×900 mm and 1400 mm×50 mm×8 mm, respectively. The modeling dataset acquired was classified into two parts: (1) measuring data derived from the welding experiment; (2) finite element analysis (FEA) data under the given welding conditions. Thus, a dataset containing 40 samples was obtained by means of physical simulation and FEA. The sample set can be separated into two disjoint sub-datasets. The learning sample set, ranging from 1st to 30th, was used for knowledge modeling (Table 3), and a test dataset composed of the remaining samples was used to examine the inference system model containing the knowledge model and the reasoning method.

As a modeling dataset is built up, the following procedure is used to deal with the data by interval-valued fuzzified treatment. Thus, the size of linguistic values for each linguistic variable, as well as the membership function of fuzzy concepts, should be provided according to the experience of specialists or welding operators.

Assume that X, Y, Z, and W are the universes of ‘leg size’, ‘current’, ‘thickness’, and ‘distortion’, respectively. Each attribute is described by several fuzzy concepts (linguistic values, usually three), and each concept corresponds to an interval-valued fuzzy subset of a universe. For convenience, all continuous attribute values are classified into several scales; e.g., the attribute value of ‘leg size’ (from 3.0 to 6.5) is discretized into eight scales, and the universe X={3.0, 3.5, …, 6.5}.

Table 3 Partial data of the welding distortion decision table*

U	Leg size (mm)	Current (A)	Thickness (mm)	Distortion (mm)
u_1	5.5	175	10	0.13
u_2	5.0	150	12	0.09
u_3	4.0	115	10	0.10
...	…	…	…	…
u_{14}	6.5	195	9	0.36
u_{15}	4.5	140	8	0.25
…	…	…	…	…
u_{29}	3.5	95	7	0.18
u_{30}	5.5	165	6	0.53

* See Table S1 for full data

Through the fuzzification treatment of Table 3, an interval-valued fuzzy decision table including attributes, linguistic values, and interval-valued fuzzy data is obtained (Table 4), in which the membership function of each interval-valued fuzzy subset is derived from the experience of experts in the welding field. In this information system, the conditional attributes a_1, a_2, and a_3 represent the leg size of the weld seam, the welding current, and the thickness of the plate, respectively, and the decision attribute d represents the minimal concave-convex deformation. Each attribute is depicted by three linguistic values; e.g., the interval-valued fuzzy subsets A_{11}, A_{12}, and A_{13} induced by a_1 are equivalent to the concepts ‘large’, ‘medium’, and ‘small’, respectively.

5.2 Rule acquisition

As per the modeling method proposed in Section 3, the procedures for attribute reduction as well as rule extraction from Table 4 are given as follows, provided that β=0.6, λ=0.8:

1. Compute the inclusion degrees of conditional classes in decision classes using Eq. (1).

Table 4 Interval-valued fuzzy decision table of welding deformation*

Attribute		u_1	u_2	u_3	…	u_{14}	u_{15}	…	u_{29}	u_{30}
	A_{11}	[0.6, 0.8]	[0.2, 0.3]	[0, 0]		[1, 1]	[0.2,0.3]		[0, 0]	[0.6, 0.8]
a_1	A_{12}	[0.2, 0.3]	[1, 1]	[0.3, 0.4]	…	[0, 0]	[1, 1]	…	[0, 0]	[0.2, 0.3]
	A_{13}	[0, 0]	[0.1, 0.2]	[0.6, 0.7]		[0, 0]	[0.1, 0.2]		[1, 1]	[0, 0]
	A_{21}	[0.6, 0.7]	[0.2, 0.2]	[0, 0]		[1, 1]	[0.2, 0.2]		[0, 0]	[0.6, 0.7]
a_2	A_{22}	[0.3, 0.3]	[1, 1]	[0.1, 0.3]	…	[0, 0]	[1, 1]	…	[0, 0]	[0.3, 0.3]
	A_{23}	[0, 0]	[0.2, 0.3]	[0.8, 0.8]		[0, 0]	[0.2, 0.3]		[1, 1]	[0, 0]
	A_{31}	[0.7, 0.8]	[1, 1]	[0.7, 0.8]		[0.1, 0.2]	[0.1, 0.2]		[0.1, 0.2]	[0, 0]
a_3	A_{32}	[0.1, 0.2]	[0, 0]	[0.1, 0.2]	…	[1, 1]	[1, 1]	…	[1, 1]	[0.2, 0.3]
	A_{33}	[0, 0]	[0, 0]	[0, 0]		[0.2, 0.2]	[0.2, 0.2]		[0.2, 0.2]	[0.7, 0.7]
	D_1	[0, 0]	[0, 0]	[0, 0]		[0.1, 0.3]	[0, 0]		[0, 0]	[0.7, 0.8]
d	D_2	[0.1, 0.2]	[0, 0]	[0, 0]	…	[1, 1]	[0.1, 0.2]	…	[0.1, 0.2]	[0.2, 0.2]
	D_3	[0.8, 0.8]	[1, 1]	[1, 1]		[0.3, 0.3]	[0.8, 0.8]		[0.8, 0.8]	[0, 0]

* See Table S2 for full data

2. Construct the interval-valued fuzzy positive region $\mathrm{POS}_B^{0.6}$ by Eq. (2).

3. Calculate the dependency degree $\gamma_B^{0.6}$ by Eq. (3).

4. Implement attribute reduction by applying the interval-valued fuzzy rough based QuickReduct algorithm (Algorithm 1).

5. Extract a fuzzy rule set from Table 4 by Eq. (4).

The above processes on rule acquisition can be achieved using MATLAB programming. The results on dependency degree, attribute reduction, and rule extraction are shown in Fig. 2. The process of attribute selection is graphically described in Fig. 3.

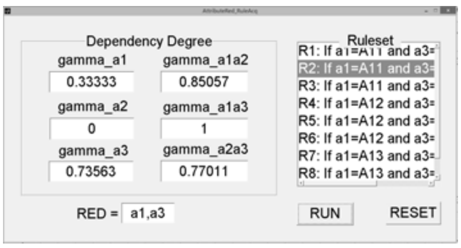

Fig. 2 Results for dependency degree, reduction, and rule extraction

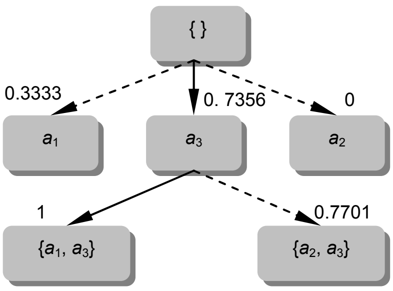

Fig. 3 Attribute selection in the IVFS decision system

5.3 Model reasoning

The fuzzy decision rule set derived from Table 4 can be represented by

$$r_p : A_1^p \times A_3^p \to D^p, \quad p = 1, 2, \cdots, 9.$$

Here $A_1^p \in \mathrm{IVFS}(X)$, $A_3^p \in \mathrm{IVFS}(Z)$, and $D^p \in \mathrm{IVFS}(Z)$.

To translate the conditional statement r_p, a ternary interval-valued fuzzy relation R_p on $X\times Y\times Z$ is constructed by an appropriate operator, where the extensional Mamdani operator is used such that

$$R_p = A_1^p \times A_3^p \times D^p, \tag{11}$$

where × denotes the Cartesian product of interval-valued fuzzy sets. As per our proposed SAR algorithm, the related inference procedures are given as follows:

1. Interval-valued fuzzify an input variable (x^*, z^*) into an interval-valued fuzzy dataset $A_1^*\times A_3^*$.

2. Translate r_p into an interval-valued fuzzy relation R_p on $X\times Z\times W$ by Eq. (11).

3. Compute the similarity grades s_p between the fact $A_1^*\times A_3^*$ and antecedent part $A_1^p\times A_3^p$ according to Eq. (6), where $s_p=\min\{S(A_1^*, A_1^p), S(A_3^*, A_3^p)\}$.

4. Modify R_p with s_p so as to generate an induced relation R_p^* according to Eq. (7), where the algebraic product is selected as a t-norm.

5. Deduce a fuzzy output D_p^* applying supremum projection on R_p^*, where

$$\mu'_{D_p^*}(w) = \sup_{x,z} \mu'_{R_p^*}(x,z,w),\ \mu''_{D_p^*}(w) = \sup_{x,z} \mu''_{R_p^*}(x,z,w).$$

6. Compute the general result using $D^*=\cup_p D_p^*$.

7. Defuzzify the interval-valued fuzzy data D^* to a specific value using the weighted means method, i.e.,

$$w^* = \sum_i \Phi_{D^*}(w_i)\cdot w_i \Big/ \sum_i \Phi_{D^*}(w_i).$$

Applying the MATLAB programming tool, an approximation inference system based on SAR is constructed, by which we can obtain the prediction values of welding deformation. The running interface of the system is shown in Fig. 4.

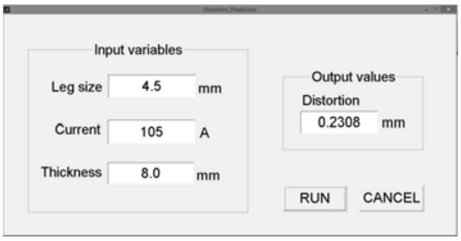

Fig. 4 Running interface for welding deformation prediction for steel structure

To inspect the effectiveness of the inference system modeled above, a welding experiment was conducted using the test samples provided in Table 5. For example, given a leg size, welding current, and plate thickness equal to 4.5 mm, 105 A, and 8.0 mm, respectively, the predicted deformation value is 0.2308 mm (Fig. 4). The prediction value, real value, and error can be represented as in Fig. 5, where 'prediction 1' and 'error 1' denote the predicted value and the error, respectively, between the predicted value and the real value. According to the 'error 1' shown in Fig. 5, the maximum error, mean error, and standard error are $E_{\max}^{(1)}$=0.0348 mm, $E_{\mathrm{m}}^{(1)}$=0.017 mm, $E_{\mathrm{std}}^{(1)}$=0.0096 mm, respectively. In terms of process modeling of welding deformation of a marine structure, the test samples are in good accordance with the knowledge model acquired, although they are not used as modeling samples.

It is well-known that the compositional rule of inference (CRI) technique is widely applied in various fields. Based on conventional CRI, the procedures related to modeling an approximate inference system for welding distortion are summarized as follows:

Table 5 Data of test samples*

Test sample	Leg size (mm)	Current (A)	Thickness (mm)	Distortion (mm)
t_1	5.5	190	6	0.53
t_2	4.5	150	8	0.26
…	…	…	…	…
t_{10}	4.5	140	5	0.46

* See Table S3 for full data

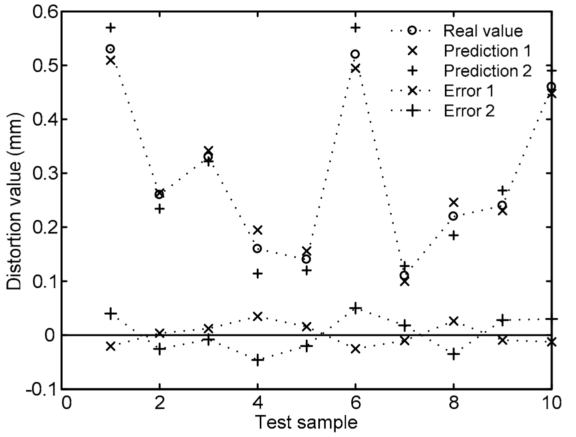

Fig. 5 Predicted values, real values, and errors of welding distortion

1. Interval-valued fuzzify an input variable (x^*, z^*) into an interval-valued fuzzy dataset $A_1^*\times A_3^*$.

2. Translate r_p into an interval-valued fuzzy relation R_p on $X\times Z\times W$ by Eq. (11).

3. Calculate a general relation R by union operation, i.e., $R=\cup_p R_p$.

4. Compute an output by $D^*=(A_1^*\times A_3^*)\circ R$, where '∘' denotes the compositional operator, and here we take sup-min operation.

5. Interval-valued defuzzify D^* to a specific value using the weighted means method.

By means of MATLAB programming, the prediction result based on the CRI approach is deduced, as 'prediction 2' (Fig. 5), where the same knowledge model and test sample set were used. According to the error between the predicted value and the real value, 'error 2' in Fig. 5, the maximum error, mean error, and standard error are $E_{\max}^{(2)}$=0.05 mm, $E_{\mathrm{m}}^{(2)}$= 0.03 mm, and $E_{\mathrm{std}}^{(2)}$=0.0131 mm, respectively.

From the prediction results shown in Fig. 5, we can see that based on the same rule set, the inference precision of the proposed SAR schema is higher than that of the conventional CRI method. According to

CRI, conclusion B^* can be deduced by $B^*=A^*\circ R=A^*\circ(A\times B)$, where R is constructed with the Mamdani operator. If $A^*\supseteq A$ and A is normal, we obtain $B^*=B$. Thus, given a conditional statement and a fact, there might be no changes in the consequent part, even if the fact is very dissimilar to the antecedent part. For the proposed inference schema, a modification of R with similarity degree s is necessary for generating an induced relation R^* and therefore, if only $A^*\neq A$ then $R^*\neq R$, i.e., $B^*\neq B$. Moreover, from Eq. (10) we see that the closer A^* is to A, the closer B^* is to B. Apparently, the inference result is reasonable because every change in a fact or an antecedent part will result in corresponding changes in a consequent part.

6 Conclusions

Knowledge acquisition is a key to the development of intelligence systems. Based on interval-valued fuzzy rough set, in this paper we provide a knowledge modeling method containing procedures including data collection, interval-valued fuzzification, and attribute reduction. By defining an inclusion based interval-valued fuzzy rough approximation, a relative positive region, and a dependency function, a simplified fuzzy knowledge model can be obtained from an interval-valued fuzzy information system. To improve the inference mechanism, we developed an approach of similarity inference by merging the techniques of CRI into the SAR method. An application case involving the prediction of marine welding deformation was used to illustrate the effectiveness of our proposed methods concerning knowledge modeling and approximation inference. The welding experimental results indicated that a higher prediction precision can be achieved using the modeling and inference methods.

In shipbuilding technology, there are some typical complex technological systems, such as welding parameter design, welding formation prediction, and heating-line formation processes of marine structures, which can be modeled using extensional rough set theory. Thus, there are many research opportunities for future applications of knowledge modeling and similarity-based inference.

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List of supplementary materials

Table S1 Full data for Table 3
Table S2 Full data for Table 4
Table S3 Full data for Table 5